CHAPTER

6 More About Triangles

What You'll Learn

Key Ideas

- Identify and construct special segments in triangles. *(Lessons 6–1 to 6–3)*
- Identify and use properties of isosceles triangles. *(Lesson 6–4)*
- Use tests for congruence of right triangles. *(Lesson 6–5)*
- Use the Pythagorean Theorem and its converse. *(Lesson 6–6)*
- Find the distance between two points on the coordinate plane. *(Lesson 6–7)*

Key Vocabulary

hypotenuse *(p. 251)*

leg *(p. 251)*

Pythagorean Theorem *(p. 256)*

Why It's Important

Construction There are reports that ancient Egyptian surveyors used a rope tool to lay out right triangles. They would do this to restore property lines that were washed away by the annual flooding of the Nile River. The need to manage their property led them to develop a mathematical tool.

Right triangles are often used in modern construction. You will investigate a right triangular tool called a *builder's square* in Lesson 6–5.

Study these lessons to improve your skills.

✓ Check Your Readiness

✓ Lesson 5–1, pp. 188–192

Classify each triangle by its angles and sides.

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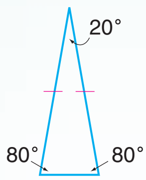

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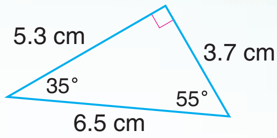

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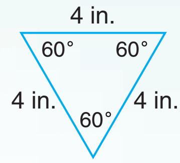
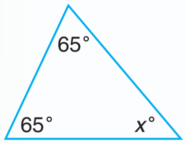
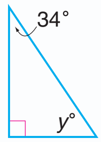
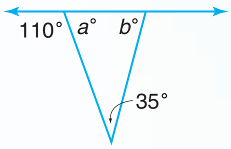


✓ Lesson 5–2, pp. 193–197

Find the value of each variable.

4.


5.


6.


✓ Algebra Review, p. 723

Solve each equation. Check your solution.

7. $7b + 1 = 50$

8. $10k - 4 = 56$

9. $6r - 8 = 2r$

10. $2n + 1 = n + 6$

11. $8x - 1 = 7x + 9$

12. $15c + 1 = 16c - 6$

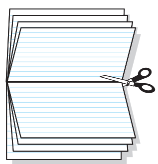
13. $12y - 3 = 8y + 13$

14. $9f + 11 = 14f + 1$

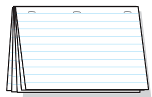
FOLDABLES™ Study Organizer

Make this Foldable to help you organize your Chapter 6 notes. Begin with four sheets of notebook paper.

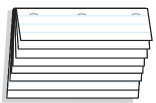
❶ **Fold** each sheet in half along the width. Then cut along the crease.



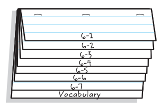
❷ **Staple** the eight half-sheets together to form a booklet.



❸ **Cut** seven lines from the bottom of the top sheet, six from the second sheet, and so on.



❹ **Label** each tab with a lesson number. The last tab is for vocabulary.



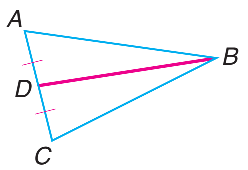
Reading and Writing As you read and study the chapter, use each page to write the main ideas, theorems, and examples for each lesson.

6–1 Medians

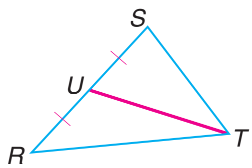
What You'll Learn
You'll learn to identify and construct medians in triangles.

Why It's Important
Travel Medians can be used to find the distance between two places.
See Exercise 22.

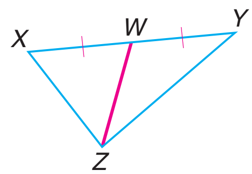
In a triangle, a **median** is a segment that joins a vertex of the triangle and the midpoint of the side opposite that vertex. In the figures below, a median of each triangle is shown in red.



median $\overline{BD}$



median $\overline{TU}$



median $\overline{ZW}$

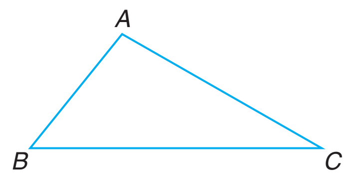
A triangle has three medians. You can use a compass and a straightedge to construct a median of a triangle.

Look Back
Construct the Bisector of a Segment: Lesson 2–3

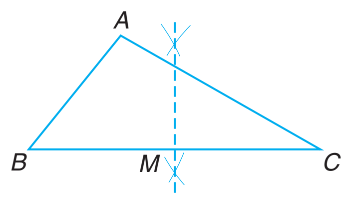
Hands-On Geometry
Construction

Materials:  compass  straightedge

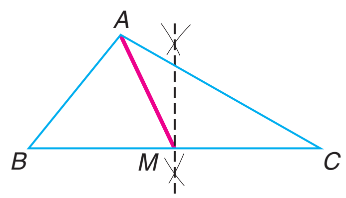
Step 1 Draw a triangle like $\triangle ABC$.



Step 2 The side opposite vertex A is $\overline{BC}$. Find the midpoint of $\overline{BC}$ by constructing the bisector of $\overline{BC}$. Label the midpoint M.



Step 3 Use a straightedge to draw $\overline{AM}$. $\overline{AM}$ is the median of $\triangle ABC$ drawn from vertex A.

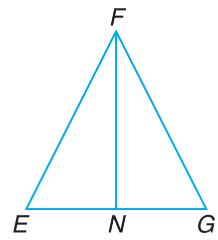


Try These

1. Draw another triangle ABC. Construct the median of $\triangle ABC$ from vertex B.
2. Construct the median of $\triangle ABC$ from vertex C.
3. Are there any other medians that can be drawn?
4. How many medians does a triangle have?

Examples

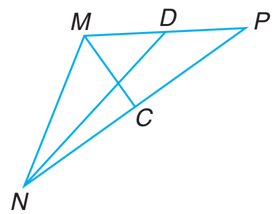
1 **In $\triangle EFG$, $\overline{FN}$ is a median. Find EN if $EG = 11$.**



$\overline{FN}$ is a median. So, N is the midpoint of $\overline{EG}$. Since $EG = 11$, $EN = \frac{1}{2} \cdot 11$ or 5.5.

Your Turn

In $\triangle MNP$, $\overline{MC}$ and $\overline{ND}$ are medians.

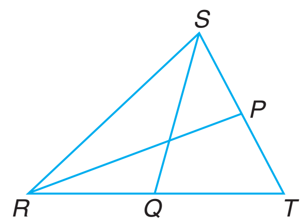


a. What is NC if $NP = 18$?

b. If $DP = 7.5$, find MP.

Algebra Link

2 **In $\triangle RST$, $\overline{RP}$ and $\overline{SQ}$ are medians. If $RQ = 7x - 1$, $SP = 5x - 4$, and $QT = 6x + 9$, find PT.**



Since $\overline{RP}$ and $\overline{SQ}$ are medians, Q and P are midpoints. Use the given values for RQ and QT to first solve for x.

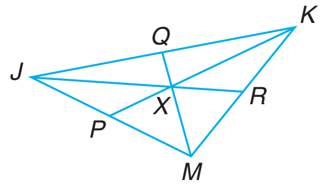
Algebra Review
Solving Multi-Step Equations, p. 723

$RQ = QT$	*Definition of Median*
$7x - 1 = 6x + 9$	*Substitution*
$7x - 1 + 1 = 6x + 9 + 1$	*Add 1 to each side.*
$7x = 6x + 10$	*Simplify.*
$7x - 6x = 6x + 10 - 6x$	*Subtract 6x from each side.*
$x = 10$	*Simplify.*

Next, use the values of x and SP to find PT.

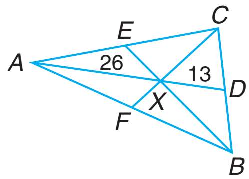
$SP = PT$	*Definition of Median*
$5x - 4 = PT$	*Substitution*
$5(10) - 4 = PT$	*Replace x with 10.*
$46 = PT$	*Simplify.*

The medians of triangle JKM, $\overline{JR}$, $\overline{KP}$, and $\overline{MQ}$, intersect at a common point called the **centroid**. When three or more lines or segments meet at the same point, the lines are **concurrent**.

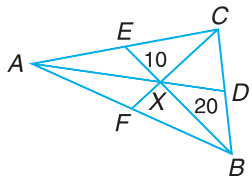


X is the centroid of $\triangle JKM$.
$\overline{JR}$, $\overline{KP}$, and $\overline{MQ}$ are concurrent.

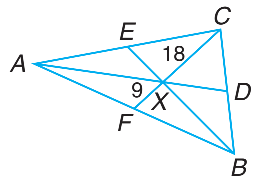
There is a special relationship between the length of the segment from the vertex to the centroid and the length of the segment from the centroid to the midpoint. Use the following diagrams to make a conjecture about the relationship between the two lengths.



$AX = 26$
$XD = 13$



$BX = 20$
$XE = 10$

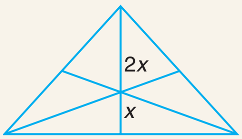


$CX = 18$
$XF = 9$

Theorem 6–1

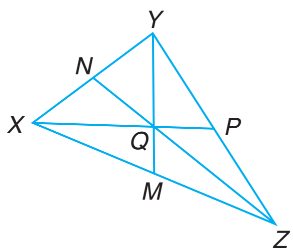
Words: The length of the segment from the vertex to the centroid is twice the length of the segment from the centroid to the midpoint.

Model:



Examples

In $\triangle XYZ$, $\overline{XP}$, $\overline{ZN}$, and $\overline{YM}$ are medians.



3 **Find ZQ if $QN = 5$.**

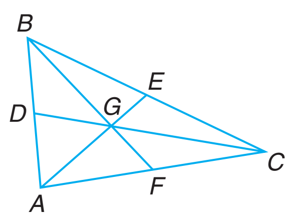
Since $QN = 5$, $ZQ = 2 \cdot 5$ or 10.

4 **If $XP = 10.5$, what is QP?**

Since $XP = 10.5$, $QP = \frac{1}{3} \cdot 10.5$ or 3.5.

Your Turn

In $\triangle ABC$, $\overline{CD}$, $\overline{BF}$, and $\overline{AE}$ are medians.



c. If $CG = 14$, what is DG?
d. Find the measure of $\overline{BF}$ if $GF = 6.8$.

Check for Understanding

Communicating Mathematics

1. **Explain** how to draw a median of a triangle.

2. **Draw** a figure that shows three concurrent segments.

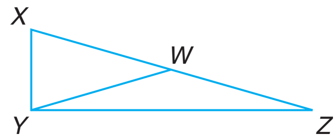
3. **You Decide?** Kim says that the medians of a triangle are always the same length. Hector says that they are never the same length. Who is correct? Explain your reasoning.

> **Vocabulary**
> median
> centroid
> concurrent

Guided Practice

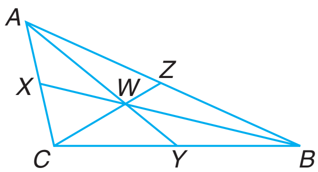
Example 1

4. In $\triangle XYZ$, $\overline{YW}$ is a median. What is XW if $XZ = 17$?



Example 2

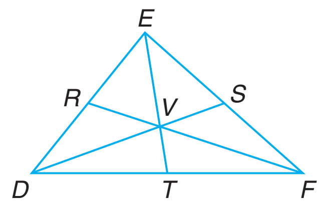
5. **Algebra** In $\triangle ABC$, $\overline{BX}$, $\overline{CZ}$, and $\overline{AY}$ are medians. If $AX = 3x - 9$, $XC = 2x - 4$, and $ZB = 2x + 1$, what is AZ?



Examples 3 & 4

In $\triangle DEF$, $\overline{DS}$, $\overline{FR}$, and $\overline{ET}$ are medians.

6. Find EV if $VT = 5$.
7. If $FR = 20.1$, what is the measure of $\overline{VR}$?

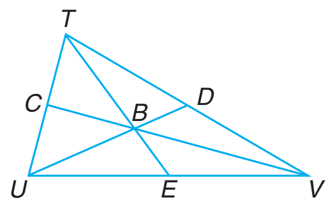


Exercises

Practice

In $\triangle TUV$, $\overline{TE}$, $\overline{UD}$, and $\overline{VC}$ are medians.

8. Find EV if $UV = 24$.
9. If $TC = 8$, find TU.
10. What is TD if $TV = 29$?



Homework Help

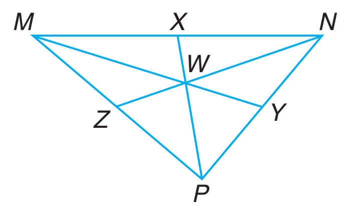
For Exercises	See Examples
8–18, 22	3, 4
20, 21	2

Extra Practice

See page 736.

In $\triangle MNP$, $\overline{MY}$, $\overline{PX}$, and $\overline{NZ}$ are medians.

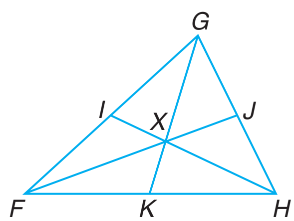
11. Find the measure of $\overline{WY}$ if $MW = 22$.
12. What is NW if $ZW = 10$?
13. If $PW = 13$, what is WX?



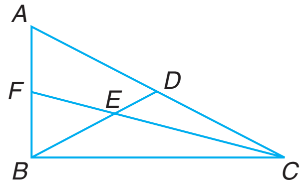
In $\triangle FGH$, $\overline{FJ}$, $\overline{HI}$, and $\overline{GK}$ are medians.

14. What is XK if $GK = 13.5$?
15. If $FX = 10.6$, what is the measure of $\overline{XJ}$?

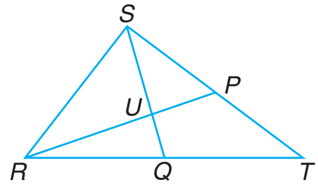
16. Find HX if $HI = 9$.



17. If $\overline{BD}$ and $\overline{CF}$ are medians of $\triangle ABC$ and $CE = 17$, what is EF?



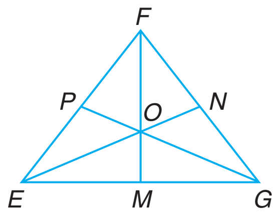
18. In $\triangle RST$, $\overline{RP}$ and $\overline{SQ}$ are medians. Find RU if $UP = 7.3$.



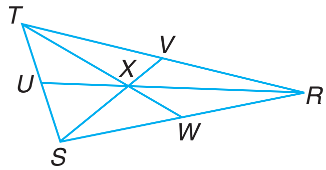
19. **Draw** a triangle with vertices R, S, and T. Then construct the medians of the triangle to show that they are concurrent.

Applications and Problem Solving

20. **Algebra** In $\triangle EFG$, $\overline{GP}$, $\overline{FM}$, and $\overline{EN}$ are medians. If $EM = 2x + 3$ and $MG = x + 5$, what is x?



21. **Algebra** $\overline{RU}$, $\overline{SV}$, and $\overline{TW}$ are medians of $\triangle RST$. What is the measure of $\overline{RW}$ if $RV = 4x + 3$, $WS = 5x - 1$, and $VT = 2x + 9$?

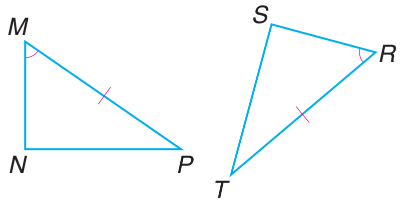


22. **Travel** On a map, the points representing the towns of Sandersville, Waynesboro, and Anderson form a triangle. The point representing Thomson is the centroid of the triangle. Suppose Washington is halfway between Anderson and Sandersville, Louisville is halfway between Sandersville and Waynesboro, and the distance from Anderson to Thomson is 75 miles. What is the distance from Thomson to Louisville?

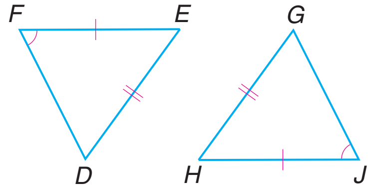
23. **Critical Thinking** Draw a triangle on a piece of cardboard and cut it out. Draw only one median on the cardboard. How can you find the centroid without using the other two medians? Place the point of a pencil on the centroid you found. Does the triangle balance on your pencil? Why?

Mixed Review

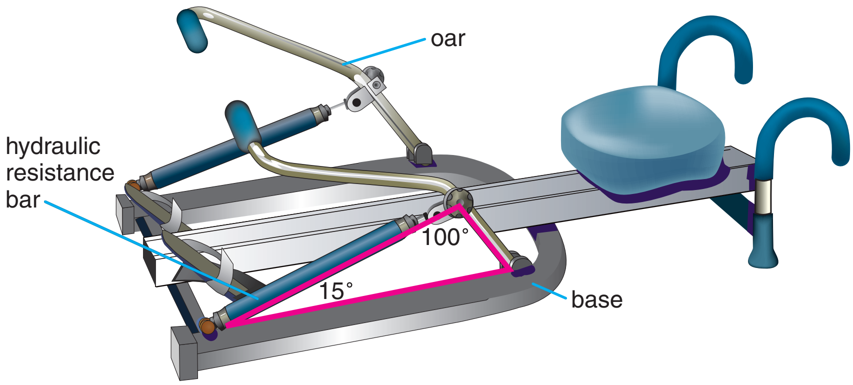
24. In $\triangle MNP$ and $\triangle RST$, $\angle M \cong \angle R$ and $\overline{MP} \cong \overline{RT}$. Name the additional congruent angles needed to show that the triangles are congruent by ASA. *(Lesson 5–6)*



25. In $\triangle DEF$ and $\triangle GHJ$, $\overline{DE} \cong \overline{GH}$, $\overline{EF} \cong \overline{HJ}$, and $\angle F \cong \angle J$. Tell whether the triangles are congruent by SAS. Explain. *(Lesson 5–5)*



26. **Fitness** A rower is designed to simulate rowing. In the diagram shown, notice that the base of the rower, along with the lower portion of the oar, and the hydraulic resistance bar form a triangle. Suppose the measures of two of the angles are 15 and 100. What is the measure of the third angle? *(Lesson 5–2)*

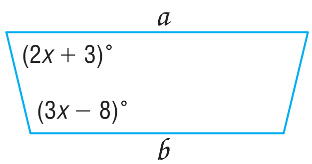


27. **Entertainment** In the equation $y = 5x + 3$, y represents the total cost of a trip to the aquarium for x people in a car. Name the slope and y-intercept of the graph of the equation and explain what each value represents. *(Lesson 4–6)*

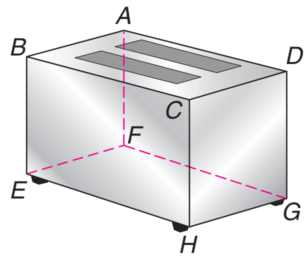
Standardized Test Practice

A B C D

28. **Grid In** Find x so that $a \parallel b$. *(Lesson 4–4)*



29. **Multiple Choice** The toaster shown at the right is formed by a series of intersecting planes. Which names a pair of skew segments? *(Lesson 4–1)*



(A) $\overline{BC}$ and $\overline{EF}$ (B) $\overline{BE}$ and $\overline{CH}$
(C) $\overline{CD}$ and $\overline{DG}$ (D) $\overline{EH}$ and $\overline{AD}$

6–2 Altitudes and Perpendicular Bisectors

What You'll Learn

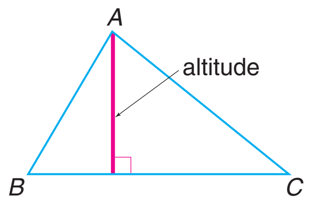
You'll learn to identify and construct altitudes and perpendicular bisectors in triangles.

Why It's Important

Construction

Carpenters use perpendicular bisectors and altitudes when framing roofs. *See Exercise 22.*

In geometry, an **altitude** of a triangle is a perpendicular segment with one endpoint at a vertex and the other endpoint on the side opposite that vertex. The altitude $\overline{AD}$ is perpendicular to side $\overline{BC}$.

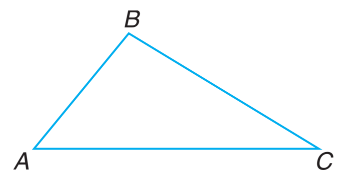


In the following activity, you will construct an altitude of a triangle.

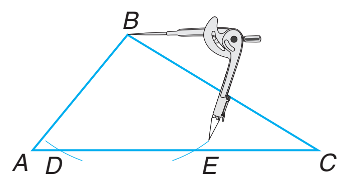
Hands-On Geometry
Construction

Materials: compass straightedge

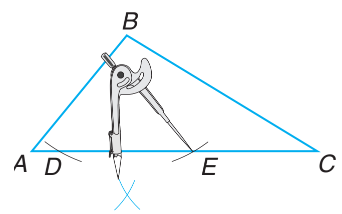
Step 1 Draw a triangle like $\triangle ABC$.



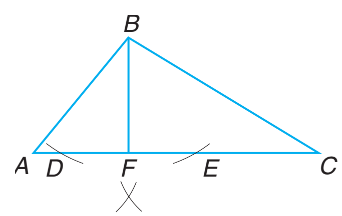
Step 2 Place the compass point at B and draw an arc that intersects side $\overline{AC}$ in two points. Label the points of intersection D and E.



Step 3 Place the compass point at D and draw an arc below $\overline{AC}$. Using the same compass setting, place the compass point at E and draw an arc to intersect the one drawn.



Step 4 Use a straightedge to align the vertex B and the point where the two arcs intersect. Draw a segment from vertex B to side $\overline{AC}$. Label the point of intersection F.



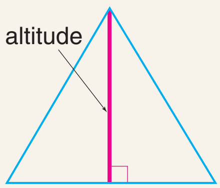
Try These

1. What can you say about $\overline{BF}$?
2. Does $\triangle ABC$ have any other altitudes? If so, construct them.
3. **Make a conjecture** about the number of altitudes in a triangle.

An altitude of a triangle may not always lie inside the triangle.

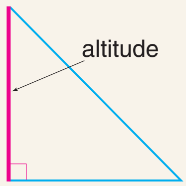
Altitudes of Triangles

acute triangle



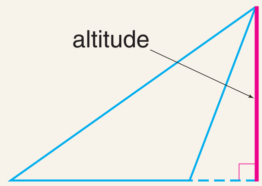
The altitude is inside the triangle.

right triangle



The altitude is a side of the triangle.

obtuse triangle

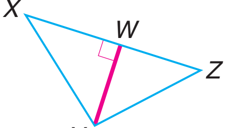


The altitude is outside the triangle.

Examples

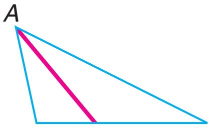
Tell whether each red segment is an altitude of the triangle.

1



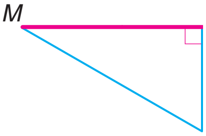
$\overline{YW} \perp \overline{XZ}$, Y is a vertex, and W is on the side opposite Y. So, $\overline{YW}$ is an altitude of the triangle.

2



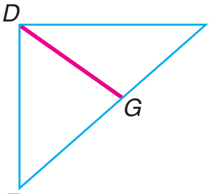
$\overline{AD}$ is not a perpendicular segment. So, $\overline{AD}$ is *not* an altitude of the triangle.

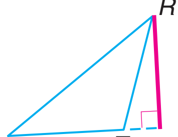
3

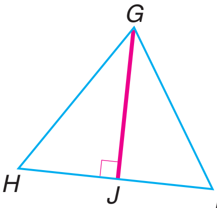


$\overline{MP} \perp \overline{NP}$, M is a vertex, and P is on the side opposite M. So, $\overline{MP}$ is an altitude of the triangle.

Your Turn

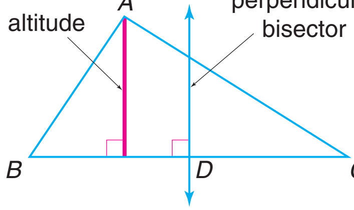
a. 

b. 

c. 

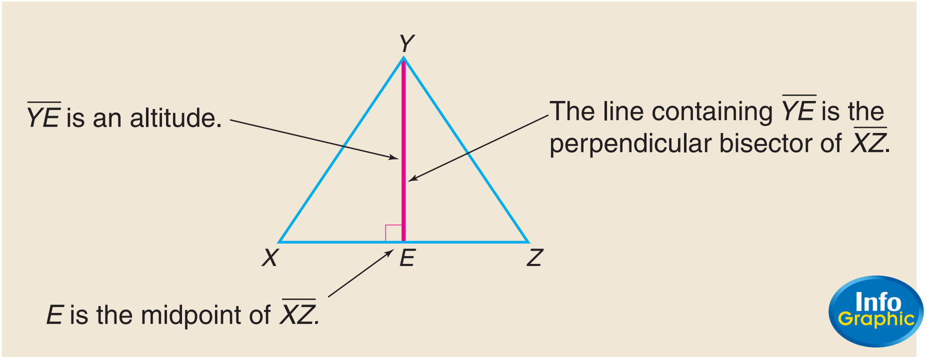
In this text, we will assume that a perpendicular bisector can be a line or a segment contained in that line.

Another special line in a triangle is a perpendicular bisector. A perpendicular line or segment that bisects a side of a triangle is called the **perpendicular bisector** of that side. Line m is a perpendicular bisector of side $\overline{BC}$.



D is the midpoint of $\overline{BC}$.

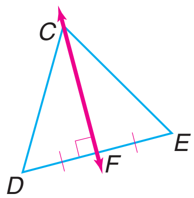
In some triangles, the perpendicular bisector and the altitude are the same. If the perpendicular bisector of a side contains the opposite vertex, then the perpendicular bisector is also an altitude.



Examples

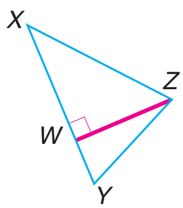
Tell whether each red line or segment is a perpendicular bisector of a side of the triangle.

4



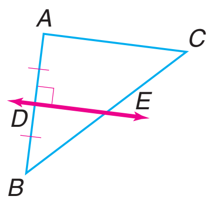
$\overleftrightarrow{CF} \perp \overline{DE}$ and F is the midpoint of $\overline{DE}$. So, $\overleftrightarrow{CF}$ is a perpendicular bisector of side $\overline{DE}$ in $\triangle CDE$.

5



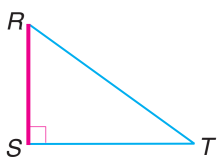
$\overline{ZW} \perp \overline{XY}$ but W is *not* the midpoint of $\overline{XY}$. So, $\overline{ZW}$ is *not* a perpendicular bisector of side $\overline{XY}$ in $\triangle XYZ$.

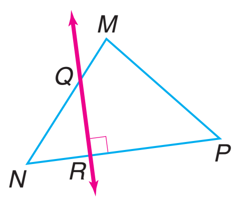
6

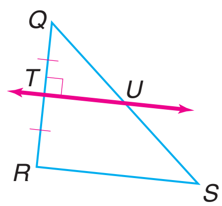


$\overleftrightarrow{DE} \perp \overline{AB}$ and D is the midpoint of $\overline{AB}$. So, $\overleftrightarrow{DE}$ is a perpendicular bisector of side $\overline{AB}$ in $\triangle ABC$.

Your Turn

d. 

e. 

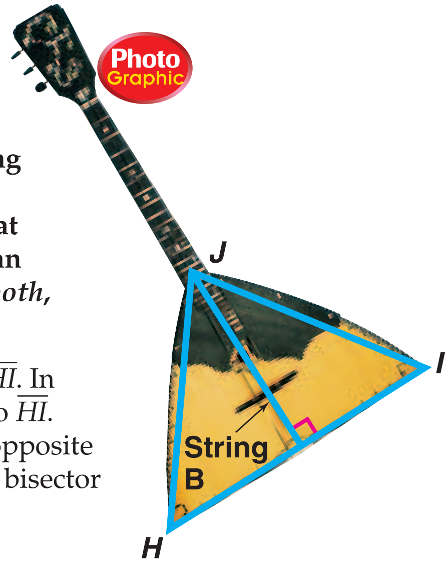
f. 

Real World

Example 7

Music Link

A balalaika is a stringed musical instrument that has a triangular body. Balalaikas are commonly played when performing Russian songs and dance music. A three-stringed balalaika is shown at the right. Tell whether string B is an *altitude*, a *perpendicular bisector*, *both*, or *neither*.



String B contains the midpoint of $\overline{HI}$. In addition, string B is perpendicular to $\overline{HI}$. Since it also contains the vertex, J, opposite $\overline{HI}$, string B is both a perpendicular bisector and an altitude.

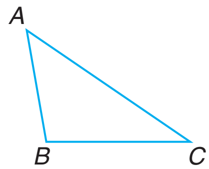
Check for Understanding

Communicating Mathematics

1. **Draw** a right triangle. Then construct all of the altitudes of the triangle.

Vocabulary
altitude
perpendicular bisector

2. **Draw** a triangle like $\triangle ABC$. Then use a segment bisector construction to construct the perpendicular bisector of $\overline{AC}$.



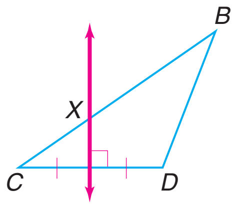
Exercise 2

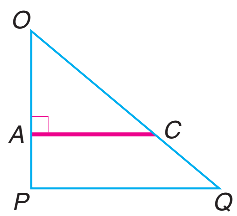
3. Writing Math Compare and contrast altitudes and perpendicular bisectors.

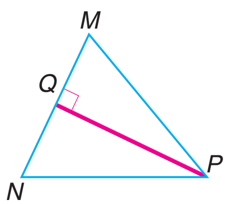
Guided Practice

For each triangle, tell whether the red segment or line is an *altitude*, a *perpendicular bisector*, *both*, or *neither*.

Examples 1–6

4. 

5. 

6. 

Example 7

7. **Camping** The front of a pup tent is shaped like a triangle. Tell whether the roof pole is an *altitude*, a *perpendicular bisector*, *both*, or *neither*.

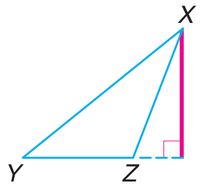
Exercises

Practice

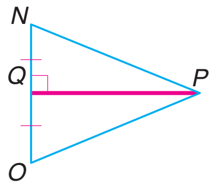
Homework Help	
For Exercises	See Examples
8–16, 18 21–23	1–6
17, 19, 20	4–6
Extra Practice	
See page 736.	

For each triangle, tell whether the red segment or line is an *altitude*, a *perpendicular bisector*, *both*, or *neither*.

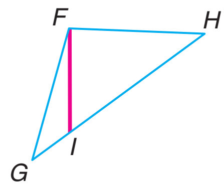
8.



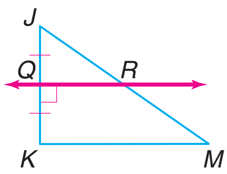
9.



10.



11.



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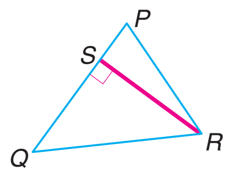
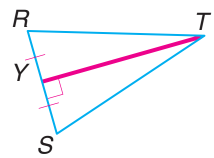
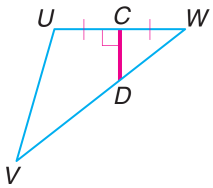


13.



14.



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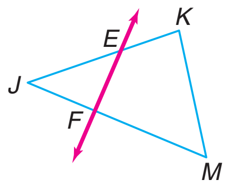
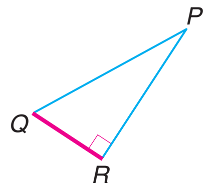
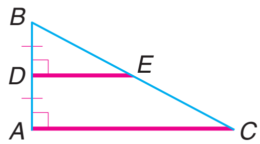


16.



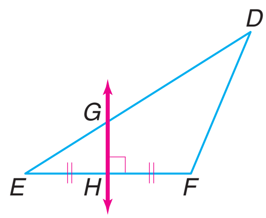
17. Name a perpendicular bisector in $\triangle ABC$.

18. Tell whether $\overline{AC}$ is a *perpendicular bisector*, an *altitude*, *both*, or *neither*.



Exercises 17–18

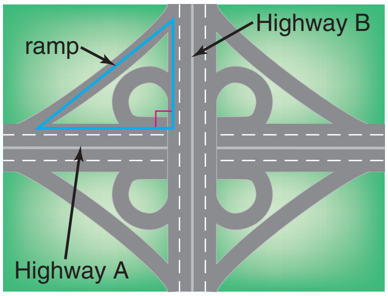
19. In $\triangle DEF$, $\overleftrightarrow{GH}$ is the perpendicular bisector of $\overline{EF}$. Is it possible to construct other perpendicular bisectors in $\triangle DEF$? Make a conjecture about the number of perpendicular bisectors of a triangle.



Applications and Problem Solving

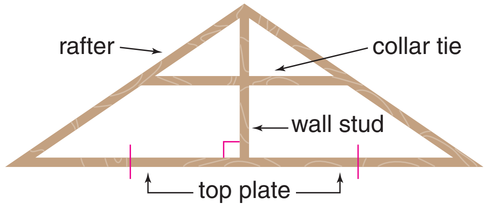
20. **Architecture** The Transamerica building in San Francisco is triangular in shape. Copy the triangle onto a sheet of paper. Then construct the perpendicular bisector of each side.

Transamerica Building



21. **Transportation** There are four major types of highway interchanges. One type, a cloverleaf interchange, is shown. Notice that each ramp along with sections of the highway form a triangle. Tell whether highway A is an *altitude*, a *perpendicular bisector*, *both*, or *neither*.

22. **Construction** The most common type of design for a house roof is a gable roof. The illustration shows the structural elements of a gable roof.



a. Which structural element is a perpendicular bisector?

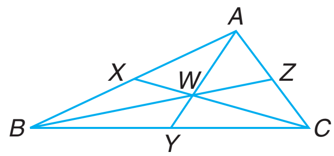
b. Tell whether the top plate is an altitude. Explain your reasoning.

c. Tell whether the collar tie is a perpendicular bisector. Explain your reasoning.

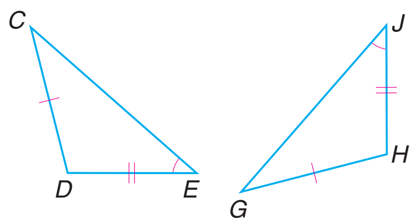
23. **Critical Thinking** Draw two types of triangles in which the altitude is on the line that forms the perpendicular bisector. Identify the types of triangles drawn, and draw the altitude and perpendicular bisector for each triangle.

Mixed Review

24. **Algebra** In $\triangle ABC$, $\overline{BZ}$, $\overline{CX}$, and $\overline{AY}$ are medians. If $BY = x - 2$ and $YC = 2x - 10$, find the value of x. *(Lesson 6–1)*



25. Determine whether $\triangle CDE$ and $\triangle GHJ$ are congruent by SSS, SAS, ASA, or AAS. If it is not possible to prove that they are congruent, write *not possible*. *(Lessons 5–5 & 5–6)*

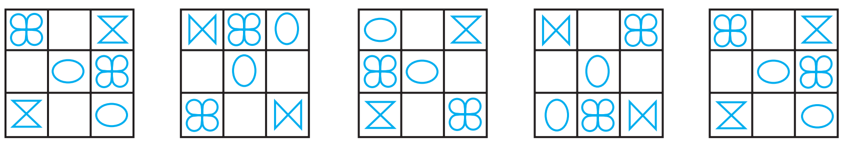


26. Find the slope of the line passing through points at (2, 3) and (−2, 4). *(Lesson 4–5)*

Standardized Test Practice

A B C D

27. **Short Response** Draw the next figure in the pattern. *(Lesson 1–1)*



28. **Multiple Choice** Andrew is buying a pair of sunglasses priced at \$18.99. What is the total cost of the sunglasses if he needs to pay a sales tax of 6%? Round to the nearest cent. *(Percent Review)*

(A) \$19.10 (B) \$19.94 (C) \$20.13 (D) \$20.54

6–3 Angle Bisectors of Triangles

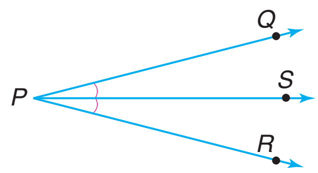
What You'll Learn

You'll learn to identify and use angle bisectors in triangles.

Why It's Important

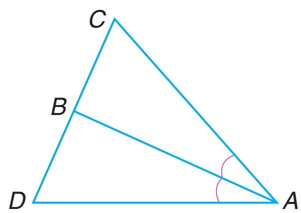
Engineering Angle bisectors of triangles can be found in bridges. *See Exercise 19.*

Recall that the bisector of an angle is a ray that separates the angle into two congruent angles.



$\overrightarrow{PS}$ bisects $\angle QPR$.
$\angle QPS \cong SPR$
$m\angle QPS = m\angle SPR$

An **angle bisector** of a triangle is a segment that separates an angle of the triangle into two congruent angles. One of the endpoints of an angle bisector is a vertex of the triangle, and the other endpoint is on the side opposite that vertex.



$\overline{AB}$ is an angle bisector of $\triangle DAC$.
$\angle DAB \cong \angle CAB$
$m\angle DAB = m\angle CAB$

Just as every triangle has three medians, three altitudes, and three perpendicular bisectors, every triangle has three angle bisectors.

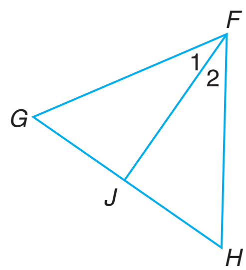
Special Segments in Triangles			
Segment	• altitude	• perpendicular bisector	• angle bisector
Type	• line segment	• line • line segment	• ray • line segment
Property	from the vertex, a line perpendicular to the opposite side	bisects the side of a triangle	bisects the angle of a triangle

An angle bisector of a triangle has all of the characteristics of any angle bisector. In $\triangle FGH$, $\overline{FJ}$ bisects $\angle GFH$.

1. $\angle 1 \cong \angle 2$, so $m\angle 1 = m\angle 2$.

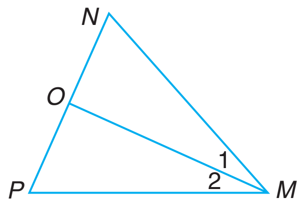
2. $m\angle 1 = \frac{1}{2}(m\angle GFH)$ or $2(m\angle 1) = m\angle GFH$

3. $m\angle 2 = \frac{1}{2}(m\angle GFH)$ or $2(m\angle 2) = m\angle GFH$



Examples

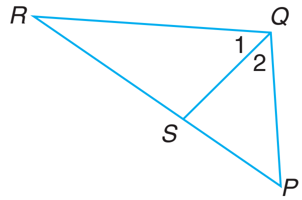
1 **In $\triangle MNP$, $\overline{MO}$ bisects $\angle NMP$. If $m\angle 1 = 33$, find $m\angle 2$.**



Since $\overline{MO}$ bisects $\angle NMP$, $m\angle 1 = m\angle 2$.

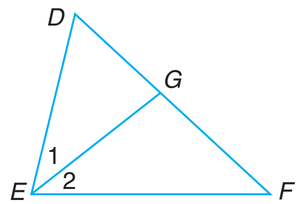
Since $m\angle 1 = 33$, $m\angle 2 = 33$.

2 **In $\triangle PQR$, $\overline{QS}$ bisects $\angle PQR$. If $m\angle PQR = 70$, what is $m\angle 2$?**



$m\angle 2 = \frac{1}{2}(m\angle PQR)$	*Definition of bisector*
$m\angle 2 = \frac{1}{2}(70)$	*Substitution*
$m\angle 2 = 35$	*Multiply.*

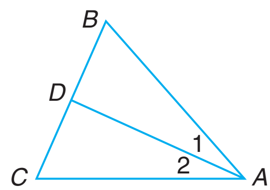
3 **In $\triangle DEF$, $\overline{EG}$ bisects $\angle DEF$. If $m\angle 1 = 43$, find $m\angle DEF$.**



$m\angle DEF = 2(m\angle 1)$	*Definition of bisector*
$m\angle DEF = 2(43)$	*Substitution*
$m\angle DEF = 86$	*Multiply.*

Your Turn

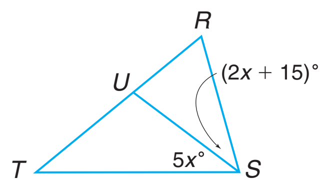
In $\triangle ABC$, $\overline{AD}$ bisects $\angle BAC$.



a. If $m\angle 1 = 32$, find $m\angle 2$.
b. Find $m\angle 1$ if $m\angle BAC = 52$.
c. What is $m\angle CAB$ if $m\angle 1 = 28$?

Algebra Link

4 **In $\triangle RST$, $\overline{SU}$ is an angle bisector. Find $m\angle UST$.**



Algebra Review
Solving Multi-Step Equations, p. 723

$m\angle UST = m\angle RSU$	*Definition of bisector*
$5x = 2x + 15$	*Substitution*
$5x - 2x = 2x + 15 - 2x$	*Subtract 2x from each side.*
$3x = 15$	*Simplify.*
$\frac{3x}{3} = \frac{15}{3}$	*Divide each side by 3.*
$x = 5$	*Simplify.*

So, $m\angle UST = 5(5)$ or 25.

Check for Understanding

Communicating Mathematics

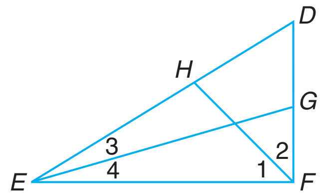
1. **Describe** an angle bisector of a triangle.
2. **Draw** an acute scalene triangle. Then use a compass and straightedge to construct the angle bisector of one of the angles.

Vocabulary
angle bisector

Guided Practice

Examples 1–3

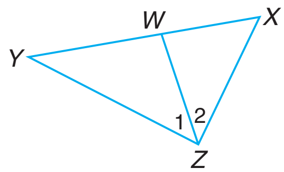
In $\triangle DEF$, $\overline{EG}$ bisects $\angle DEF$, and $\overline{FH}$ bisects $\angle EFD$.



3. If $m\angle 4 = 24$, what is $m\angle DEF$?
4. Find $m\angle 2$ if $m\angle 1 = 36$.
5. What is $m\angle EFD$ if $m\angle 1 = 42$?

Example 4

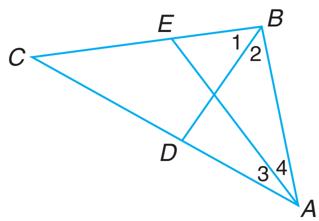
6. **Algebra** In $\triangle XYZ$, $\overline{ZW}$ bisects $\angle YZX$. If $m\angle 1 = 5x + 9$ and $m\angle 2 = 39$, find x.



Exercises

Practice

In $\triangle ABC$, $\overline{BD}$ bisects $\angle ABC$, and $\overline{AE}$ bisects $\angle BAC$.



7. If $m\angle 1 = 55$, what is $m\angle ABC$?
8. Find $m\angle 3$ if $m\angle BAC = 38$.
9. What is $m\angle 4$ if $m\angle 3 = 22$?
10. Find $m\angle 2$ if $m\angle ABC = 118$.
11. What is $m\angle BAC$ if $m\angle 3 = 20$?

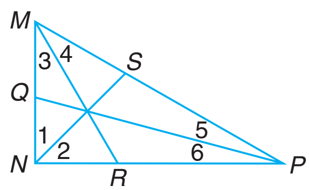
Homework Help

For Exercises	See Examples
7, 11, 15, 17	3
8, 10, 13, 14, 16, 20	2
9, 12	1
18	4
19	2, 3

Extra Practice

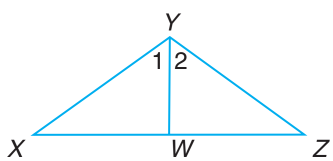
See page 736.

In $\triangle MNP$, $\overline{NS}$ bisects $\angle MNP$, $\overline{MR}$ bisects $\angle NMP$, and $\overline{PQ}$ bisects $\angle MPN$.



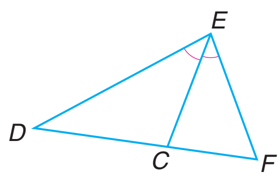
12. Find $m\angle 4$ if $m\angle 3 = 31$.
13. If $m\angle MPN = 34$, what is $m\angle 6$?
14. What is $m\angle 3$ if $m\angle NMP = 64$?
15. Find $m\angle MNP$ if $m\angle 1 = 44$.
16. What is $m\angle 2$ if $\angle MNP$ is a right angle?

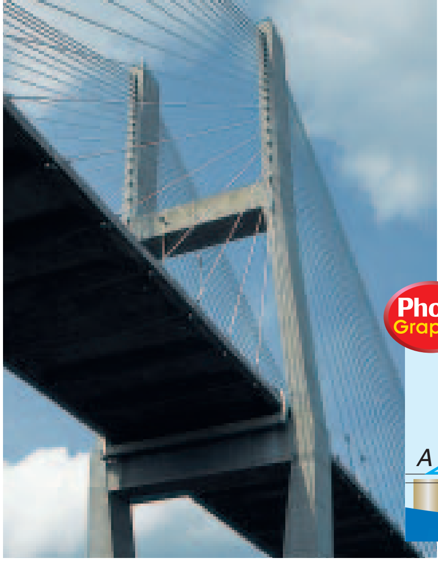
17. In $\triangle XYZ$, $\overline{YW}$ bisects $\angle XYZ$. What is $m\angle XYZ$ if $m\angle 2 = 62$?



Applications and Problem Solving

18. **Algebra** In $\triangle DEF$, $\overline{EC}$ is an angle bisector. If $m\angle CEF = 2x + 10$ and $m\angle DEC = x + 25$, find $m\angle DEC$.



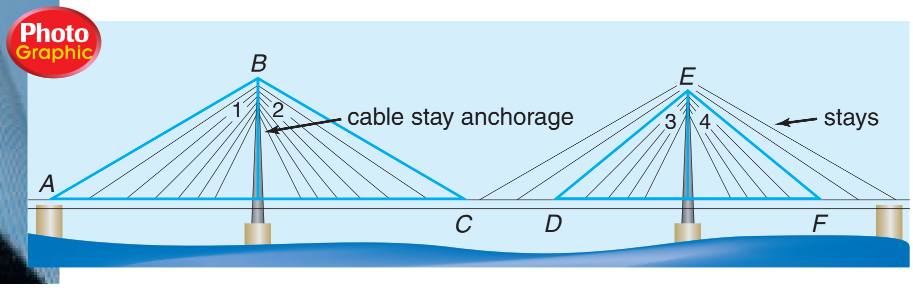


Tallmadge Bridge, Savannah, Georgia

19. **Engineering** One type of bridge, a *cable-stayed bridge* is shown. Notice that the *cable stay anchorage* is an angle bisector of each triangle formed by the cables called *stays* and the roadway.

a. Suppose $m\angle ABC = 120$, what is $m\angle 2$?

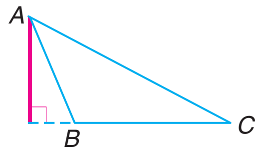
b. Suppose $m\angle 4 = 48$, what is $m\angle DEF$?



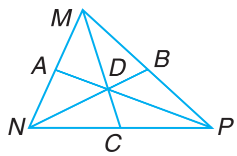
20. **Critical Thinking** What kind of angles are formed when you bisect an obtuse angle of a triangle? Explain.

Mixed Review

21. Tell whether the red segment in $\triangle ABC$ is an *altitude*, a *perpendicular bisector*, *both*, or *neither*. *(Lesson 6–2)*



22. In $\triangle MNP$, $\overline{MC}$, $\overline{NB}$, and $\overline{PA}$ are medians. Find PD if $DA = 6$. *(Lesson 6–1)*



Exercise 22

23. **Algebra** The measures of the angles of a triangle are $x + 2$, $4x + 3$, and $x + 7$. Find the measure of each angle. *(Lesson 5–2)*

Standardized Test Practice

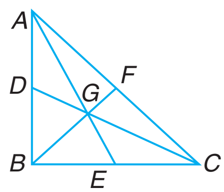
24. **Short Response** Triangle DEF has sides that measure 6 feet, 6 feet, and 9 feet. Classify the triangle by its sides. *(Lesson 5–1)*

25. **Multiple Choice** Multiply $2r + s$ by $r - 3s$. *(Algebra Review)*

(A) $2r^2 - 3s^2$ (B) $2r^2 - 5rs - 3s^2$ (C) $2r^2 - 3rs$ (D) $-6r^2s^2$

Quiz 1 Lessons 6–1 through 6–3

In $\triangle ABC$, $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ are medians. *(Lesson 6–1)*

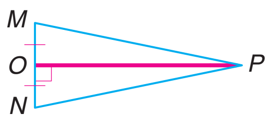


1. Find GE if $AG = 9$.

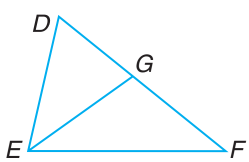
2. What is BF if $BG = 5$?

3. If $DG = 12$, what is the measure of $\overline{DC}$?

4. Tell whether $\overline{PO}$ is an *altitude*, a *perpendicular bisector*, *both*, or *neither*. *(Lesson 6–2)*



5. **Algebra** In $\triangle DEF$, $\overline{EG}$ is an angle bisector. If $m\angle DEG = 2x + 7$ and $m\angle GEF = 4x - 1$, find $m\angle GEF$. *(Lesson 6–3)*



Chapter 6 Investigation

What a CIRCLE!

Materials
- ruler
- protractor
- compass

Circumcenter, Centroid, Orthocenter, and Incenter

Is there a relationship between the perpendicular bisectors of the sides of a triangle, the medians, the altitudes, and the angle bisectors of a triangle? Let's find out!

Investigate

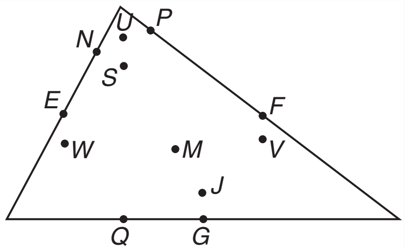
1. Use construction tools to locate some interesting points on a triangle.
   a. Draw a large acute scalene triangle.
   b. On a separate sheet of paper, copy the following table.

Description of Points	Label of Points
midpoints of the three sides (3)	
circumcenter (1)	
centroid (1)	
intersection points of altitudes with the sides (3)	
orthocenter (1)	
midpoints of segments from orthocenter to each vertex (3)	
incenter (1)	
midpoint of segment joining circumcenter and orthocenter (1)	

   c. Construct the perpendicular bisector of each side of your triangle. Label the midpoints *E*, *F*, and *G*. Record these letters in your table. The **circumcenter** is the point where the perpendicular bisectors meet. Label this point *J* and record it. To avoid confusion, erase the perpendicular bisectors, but not the circumcenter.
   d. Draw the medians of your triangle. The point where the medians meet is the *centroid*. Label this point *M* and record it in your table. Erase the medians, but not the centroid.

CONTENTS

e. Construct the altitudes of the triangle. Label the points where the altitudes intersect the sides *N*, *P*, and *Q*. Record these points. The point where the altitudes meet is the **orthocenter**. Label this point *S* and record it. Erase the altitudes, but not the orthocenter.



f. Draw three segments, each having the orthocenter as one endpoint and a vertex of your triangle as the other endpoint. Find the midpoint of each segment. Label the midpoints *U*, *V*, and *W* and record these points in your table. Erase the segments.

g. Construct the bisector of each angle of the triangle. The point where the angle bisectors meet is the **incenter**. Label this point *X* and record it. Erase the angle bisectors, but not the incenter.

2. You should now have 13 points labeled. Follow these steps to construct a special circle, called a **nine-point circle**.

   a. Locate the circumcenter and orthocenter. Draw a line segment connecting these two points. Bisect this line segment. Label the midpoint *Z* and record it in the table. Do not erase this segment.

   b. Draw a circle whose center is point *Z* and whose radius extends to a midpoint of the side of your triangle. How many of your labeled points lie on or very close to this circle?

   c. Extend the segment drawn in Step 2a. This line is called the **Euler** (OY-ler) **line**. How many points are on the Euler line?

Extending the Investigation

In this extension, you will determine whether a special circle exists for other types of triangles.

Use paper and construction tools to investigate these cases.

1. an obtuse scalene triangle **2.** a right triangle **3.** an equilateral triangle

Presenting Your Conclusions

Here are some ideas to help you present your conclusions to the class.

- Make a booklet of your constructions. For each triangle, include a table in which all of the points are recorded.
- Research Leonhard Euler. Write a brief report on his contributions to mathematics, including the nine-point circle.

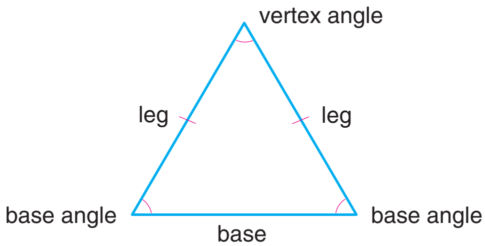
interNET CONNECTION **Investigation** For more information on the nine-point circle, visit: www.geomconcepts.com

6–4 Isosceles Triangles

What You'll Learn
You'll learn to identify and use properties of isosceles triangles.

Why It's Important
Advertising Isosceles triangles can be found in business logos. *See Exercise 17.*

Recall from Lesson 5–1 that an isosceles triangle has at least two congruent sides. The congruent sides are called **legs**. The side opposite the vertex angle is called the **base**. In an isosceles triangle, there are two base angles, the vertices where the base intersects the congruent sides.



You can use a TI–83/84 Plus graphing calculator to draw an isosceles triangle and study its properties.

Graphing Calculator Tutorial
See pp. 782–785.

Graphing Calculator Exploration

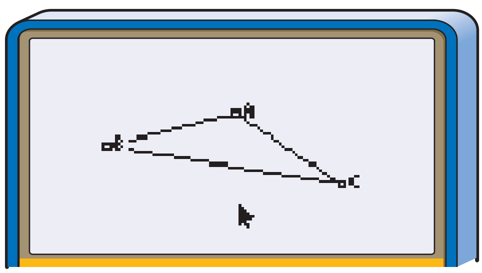
Step 1 Draw a circle using the Circle tool on the F2 menu. Label the center of the circle *A*.

Step 2 Use the Triangle tool on the F2 menu to draw a triangle that has point *A* as one vertex and its other two vertices on the circle. Label these vertices *B* and *C*.

Step 3 Use the Hide/Show tool on menu F5 to hide the circle. Press the CLEAR key to quit the F7 menu. The figure that remains on the screen is isosceles triangle *ABC*.

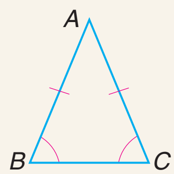
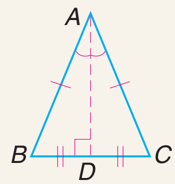
Try These

1. Tell how you can use the measurement tools on F5 to check that $\triangle ABC$ is isosceles. Use your method to be sure it works.
2. Use the Angle tool on F5 to measure $\angle B$ and $\angle C$. What is the relationship between $\angle B$ and $\angle C$?
3. Use the Angle Bisector tool on F3 to bisect $\angle A$. Use the Intersection Point tool on F2 to mark the point where the angle bisector intersects $\overline{BC}$. Label the point of intersection *D*. What is point *D* in relation to side $\overline{BC}$?



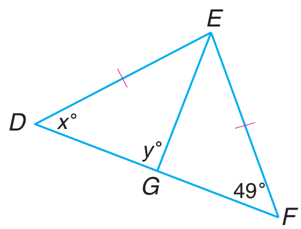
4. Use the Angle tool on F5 to find the measures of $\angle ADB$ and $\angle ADC$.
5. Use the Distance & Length tool on F5 to measure $\overline{BD}$ and $\overline{CD}$. What is the relationship between the lengths of $\overline{BD}$ and $\overline{CD}$?
6. Is $\overline{AD}$ part of the perpendicular bisector of $\overline{BC}$? Explain.

The results you found in the activity are expressed in the following theorems.

Theorem	Words	Models	Symbols
6–2 Isosceles Triangle Theorem	If two sides of a triangle are congruent, then the angles opposite those sides are congruent.		If $\overline{AB} \cong \overline{AC}$, then $\angle C \cong \angle B$.
6–3	The median from the vertex angle of an isosceles triangle lies on the perpendicular bisector of the base and the angle bisector of the vertex angle.		If $\overline{AB} \cong \overline{AC}$ and $\overline{BD} \cong \overline{CD}$, then $\overline{AD} \perp \overline{BC}$ and $\angle BAD \cong \angle CAD$.

Example 1

Find the value of each variable in isosceles triangle *DEF* if $\overline{EG}$ is an angle bisector.



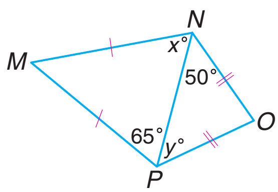
First, find the value of x.
Since $\triangle DEF$ is an isosceles triangle, $\angle D \cong \angle F$. So, $x = 49$.

Now find the value of y.
By Theorem 6–3, $\overline{EG} \perp \overline{DF}$. So, $y = 90$.

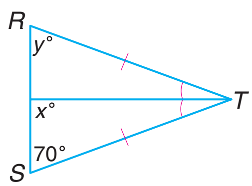
Your Turn

For each triangle, find the values of the variables.

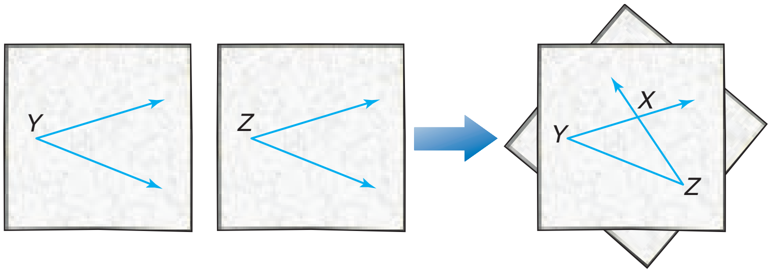
a.



b.



Suppose you draw two congruent acute angles on two pieces of patty paper and then rotate one of the angles so that one pair of rays overlaps and the other pair intersects.

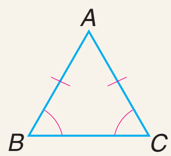


What kind of triangle is formed?
What is true about angles Y and Z?
What is true about the sides opposite angles Y and Z?
Is the converse of Theorem 6–2 true?

Theorem 6–4 Converse of Isosceles Triangle Theorem

Words: If two angles of a triangle are congruent, then the sides opposite those angles are congruent.

Model:



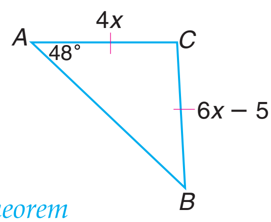
Symbols: If $\angle B \cong \angle C$, then $\overline{AC} \cong \overline{AB}$.

Example 2

Algebra Link

Algebra Review
Solving Multi-Step Equations, p. 723

In $\triangle ABC$, $\angle A \cong \angle B$ and $m\angle A = 48$. Find $m\angle C$, AC, and BC.



First, find $m\angle C$. You know that $m\angle A = 48$. Since $\angle A \cong \angle B$, $m\angle B = 48$.

$m\angle A + m\angle B + m\angle C = 180$	*Angle Sum Theorem*
$48 + 48 + m\angle C = 180$	*Replace m∠A and m∠B with 48.*
$96 + m\angle C = 180$	*Add.*
$96 - 96 + m\angle C = 180 - 96$	*Subtract 96 from each side.*
$m\angle C = 84$	*Simplify.*

Next, find AC. Since $\angle A \cong \angle B$, Theorem 6–4 states that $\overline{BC} \cong \overline{AC}$.

$BC = AC$	*Definition of Congruent Segments*
$6x - 5 = 4x$	*Replace AC with 4x and BC with 6x − 5.*
$6x - 5 - 6x = 4x - 6x$	*Subtract 6x from each side.*
$-5 = -2x$	*Simplify.*
$\frac{-5}{-2} = \frac{-2x}{-2}$	*Divide each side by −2.*
$2.5 = x$	*Simplify.*

By replacing x with 2.5, you find that $AC = 4(2.5)$ or 10 and $BC = 6(2.5) - 5$ or 10.

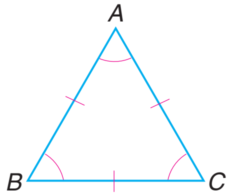
CONTENTS

Look Back

Equiangular: Lesson 5–2; Equilateral: Lesson 5–1

In Chapter 5, the terms *equiangular* and *equilateral* were defined. Using Theorem 6–4, we can now establish that equiangular triangles are equilateral.

$\triangle ABC$ is equiangular.
Since $m\angle A = m\angle B = m\angle C$,
Theorem 6–4 implies that
$BC = AC = AB$.



Theorem 6–5	A triangle is equilateral if and only if it is equiangular.

Check for Understanding

Communicating Mathematics

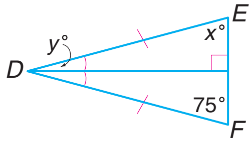
1. **Draw** an isosceles triangle. Label it $\triangle DEF$ with base $\overline{DF}$. Then state four facts about the triangle.
2. **Explain** why equilateral triangles are also equiangular and why equiangular triangles are also equilateral.

Guided Practice

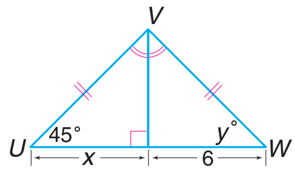
For each triangle, find the values of the variables.

Example 1

3.

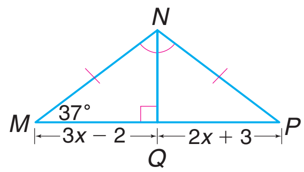


4.



Example 2

5. **Algebra** In $\triangle MNP$, $\angle M \cong \angle P$ and $m\angle M = 37$. Find $m\angle P$, MQ, and PQ.

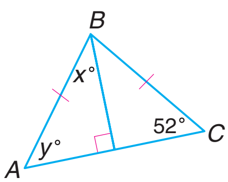


Exercises

Practice

For each triangle, find the values of the variables.

6.



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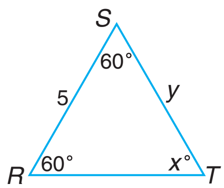

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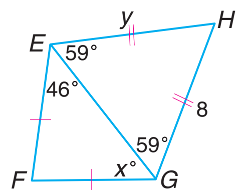
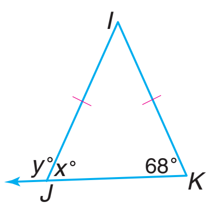


Homework Help	
For Exercises	See Examples
6-14, 17, 18	1
15, 16	2
Extra Practice	
See page 737.	

9.



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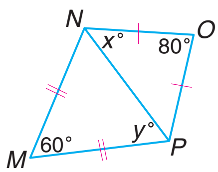

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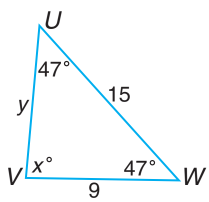
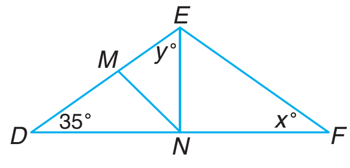


12. In $\triangle DEF$, $\overline{DE} \cong \overline{FE}$. If $m\angle D = 35$, what is the value of x?

13. Find the value of y if $\overline{EN} \perp \overline{DF}$.

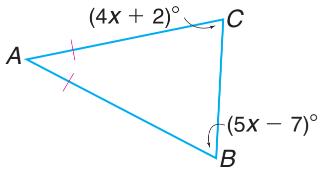
14. In $\triangle DMN$, $\overline{DM} \cong \overline{MN}$. Find $m\angle DMN$.



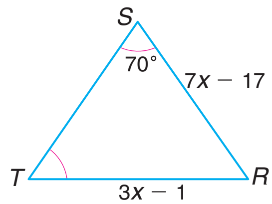
Exercises 12–14

Applications and Problem Solving

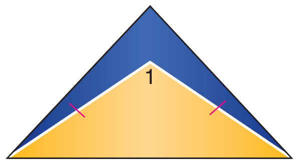
15. **Algebra** In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$. If $m\angle B = 5x - 7$ and $m\angle C = 4x + 2$, find $m\angle B$ and $m\angle C$.



16. **Algebra** In $\triangle RST$, $\angle S \cong \angle T$, $m\angle S = 70$, $RT = 3x - 1$, and $RS = 7x - 17$. Find $m\angle T$, RT, and RS.



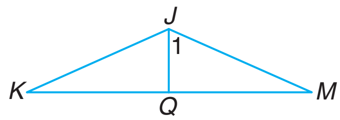
17. **Advertising** A business logo is shown.
   a. What kind of triangle does the logo contain?
   b. If the measure of angle 1 is 110, what are the measures of the two base angles of that triangle?



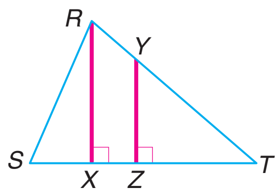
18. **Critical Thinking** Find the measures of the angles of an isosceles triangle such that, when an angle bisector is drawn, two more isosceles triangles are formed.

Mixed Review

19. In $\triangle JKM$, $\overline{JQ}$ bisects $\angle KJM$. If $m\angle KJM = 132$, what is $m\angle 1$? *(Lesson 6–3)*



20. In $\triangle RST$, $\overline{SZ} \cong \overline{TZ}$. Name a perpendicular bisector. *(Lesson 6–2)*



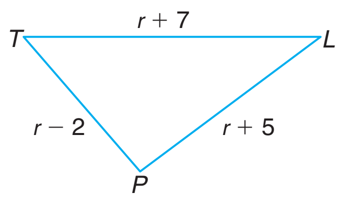
Exercise 20

21. Graph and label point H at $(-4, 3)$ on a coordinate plane. *(Lesson 2–4)*

Standardized Test Practice

A B C D

22. **Short Response** Marcus used 37 feet of fencing to enclose his triangular garden. What is the length of each side of the garden? *(Lesson 1–6)*



23. **Short Response** Write a sequence in which each term is 7 less than the previous term. *(Lesson 1–1)*

6–5 Right Triangles

What You'll Learn

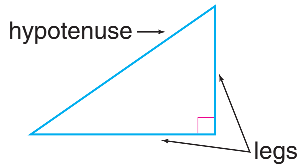
You'll learn to use tests for congruence of right triangles.

Why It's Important

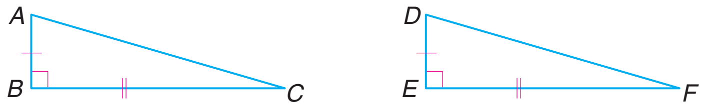
Construction

Masons use right triangles when building brick, block, and stone structures.
See Exercise 20.

In a right triangle, the side opposite the right angle is called the **hypotenuse**. The two sides that form the right angle are called the **legs**.



In triangles *ABC* and *DEF*, the two right angles are congruent. Also, the corresponding legs are congruent. So, the triangles are congruent by SAS.



Reading Geometry

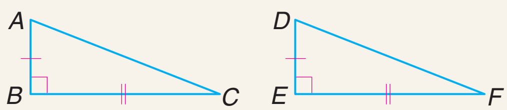
The abbreviation LL is read as *Leg-Leg*.

Since right triangles are special cases of triangles, the SAS test for congruence can be used to establish the following theorem.

Theorem 6–6 LL Theorem

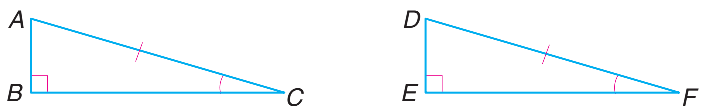
Words: If two legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.

Model:



$\triangle ABC \cong \triangle DEF$

Suppose the hypotenuse and an acute angle of the triangle on the left are congruent to the hypotenuse and acute angle of the triangle on the right.



Since the right angles in each triangle are congruent, the triangles are congruent by AAS. The AAS Theorem leads to Theorem 6–7.

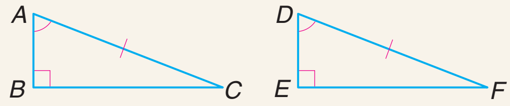
Reading Geometry

The abbreviation HA is read as *Hypotenuse-Acute Angle.*

Theorem 6–7 HA Theorem

Words: If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and corresponding angle of another right triangle, then the triangles are congruent.

Model:

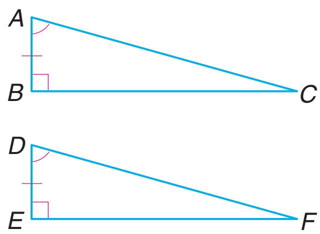


Symbols: $\triangle ABC \cong \triangle DEF$

Suppose a leg and an acute angle of one triangle are congruent to the corresponding leg and acute angle of another triangle.

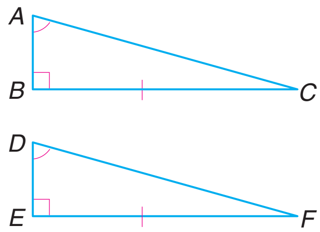
Case 1

The leg is included between the acute angle and the right angle.



Case 2

The leg is not included between the acute angle and the right angle.



The right angles in each triangle are congruent. In Case 1, the triangles are congruent by ASA. In Case 2, the triangles are congruent by AAS. This leads to Theorem 6–8.

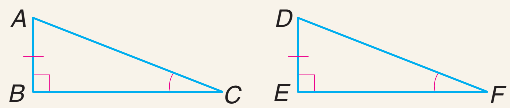
Reading Geometry

The abbreviation LA is read as *Leg-Acute Angle.*

Theorem 6–8 LA Theorem

Words: If one leg and an acute angle of a right triangle are congruent to the corresponding leg and angle of another right triangle, then the triangles are congruent.

Model:



Symbols: $\triangle ABC \cong \triangle DEF$

The following postulate describes the congruence of two right triangles when the hypotenuse and a leg of the two triangles are congruent.

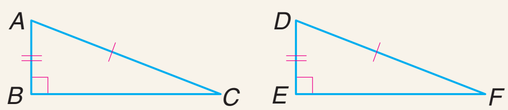
Reading Geometry

The abbreviation HL is read as *Hypotenuse-Leg.*

Postulate 6–1 HL Postulate

Words: If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.

Model:

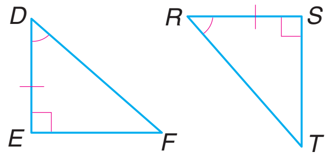


Symbols: $\triangle ABC \cong \triangle DEF$

Examples

Determine whether each pair of right triangles is congruent by LL, HA, LA, or HL. If it is not possible to prove that they are congruent, write *not possible*.

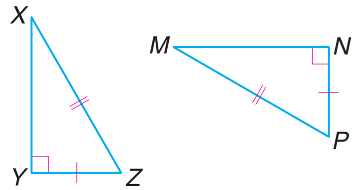
1



There is one pair of congruent acute angles, $\angle D \cong \angle R$. There is one pair of congruent legs, $\overline{DE} \cong \overline{RS}$.

So, $\triangle DEF \cong \triangle RST$ by LA.

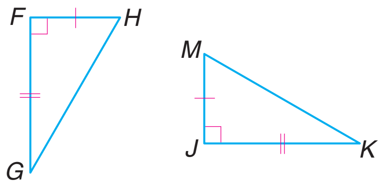
2

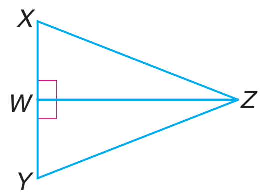


There is one pair of congruent legs, $\overline{YZ} \cong \overline{NP}$. The hypotenuses are congruent, $\overline{XZ} \cong \overline{MP}$.

So, $\triangle XYZ \cong \triangle MNP$ by HL.

Your Turn

a. 

b. 

Check for Understanding

Communicating Mathematics

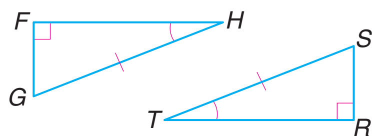
1. **Tell** which test for congruence is used to establish the LL Theorem.
2. **Write** a few sentences explaining the LL, HA, LA, and HL tests for congruence. Give an example of each.

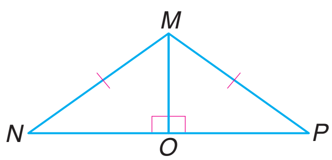
Vocabulary
hypotenuse
legs

Guided Practice

Determine whether each pair of right triangles is congruent by LL, HA, LA, or HL. If it is not possible to prove that they are congruent, write *not possible*.

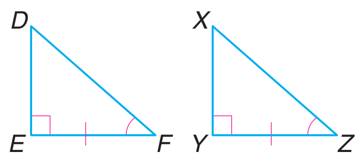
Examples 1 & 2

3. 

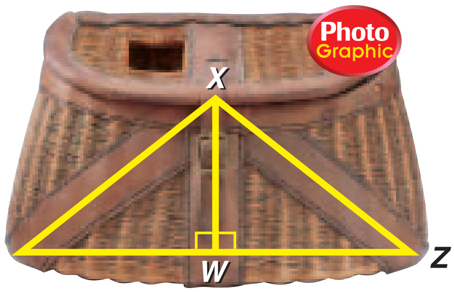
4. 

Examples 1 & 2

5. Which test for congruence proves that $\triangle DEF \cong \triangle XYZ$?



6. **Sports** A creel is a wicker basket used for holding fish. On the creel shown, the straps form two right triangles. Explain how $\triangle XYW \cong \triangle XZW$ by the HL Postulate if $\overline{XY} \cong \overline{XZ}$.

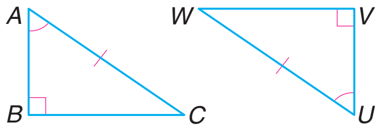


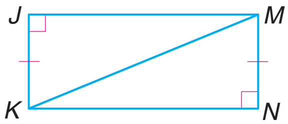
Exercises

Practice

Homework Help	
For Exercises	See Examples
7, 16, 21	1
8, 9, 15, 17, 19	2
Extra Practice	
See page 737.	

Determine whether each pair of right triangles is congruent by LL, HA, LA, or HL. If it is not possible to prove that they are congruent, write *not possible*.

7.


8.


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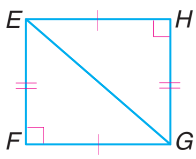
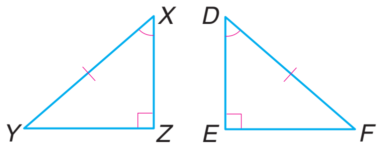
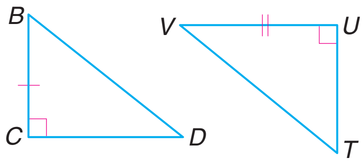
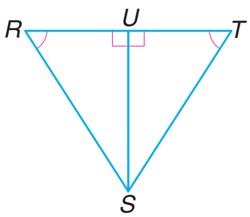
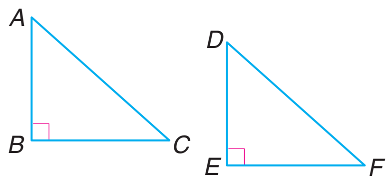


10.


11.


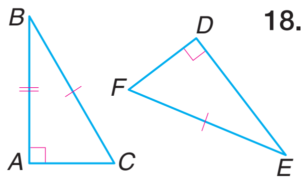
12.


Given $\triangle ABC$ and $\triangle DEF$, name the corresponding parts needed to prove that $\triangle ABC \cong \triangle DEF$ by each theorem.

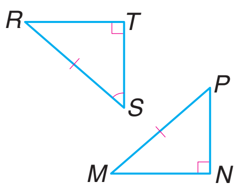


13. LL
14. HA
15. HL
16. LA

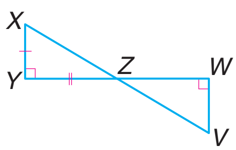
Name the corresponding parts needed to prove the triangles congruent. Then complete the congruence statement and name the theorem used.

17.


$\triangle BAC \cong \triangle$ __?__

18.


$\triangle RTS \cong \triangle$ __?__

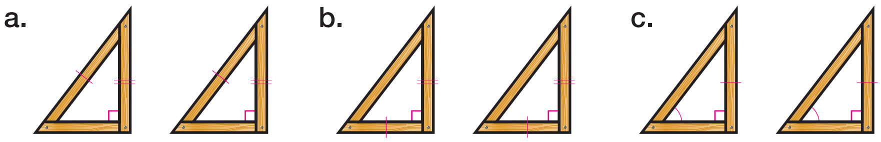
19.


$\triangle XYZ \cong \triangle$ __?__

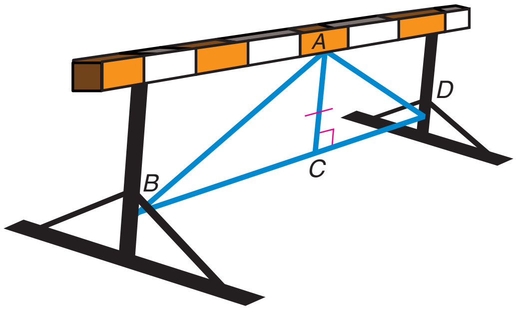
CONTENTS

Applications and Problem Solving

20. **Construction** Masons use bricks, concrete blocks, and stones to build various structures with right angles. To check that the corners are right angles, they use a tool called a *builder's square*. Which pair of builder's squares are congruent by the LL Theorem?



21. **Sports** There are many types of hurdles in track and field. One type, a steeplechase hurdle, is shown. Notice that the frame contains many triangles. In the figure, $\overline{AC}$ bisects $\angle BAD$, and $\overline{AC} \perp \overline{BD}$. What theorem can you use to prove $\triangle ABC \cong \triangle ADC$? Explain.

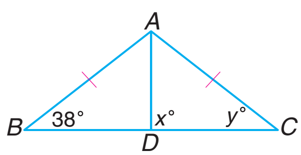


interNET CONNECTION **Data Update** For the latest information on track and field, visit: www.geomconcepts.com

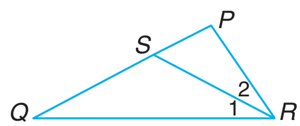
22. **Critical Thinking** Postulates SAS, ASA, and SSS require three parts of a triangle be congruent to three parts of another triangle for the triangles to be congruent. Explain why postulates LA, LL, HA, and HL require that only two parts of a triangle be congruent to two parts of another triangle in order for the triangles to be congruent.

Mixed Review

23. Find the value of each variable in triangle *ABC* if $\overline{AD}$ is an angle bisector. *(Lesson 6–4)*



24. In $\triangle PQR$, $\overline{RS}$ bisects $\angle PRQ$. If $m\angle 2 = 36$, find $m\angle PRQ$. *(Lesson 6–3)*

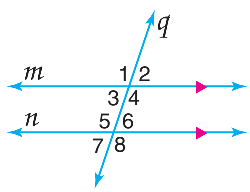


25. If $\triangle ABC \cong \triangle XYZ$, $m\angle A = 40$, and $m\angle C = 65$, find $m\angle Y$. *(Lesson 5–4)*

Standardized Test Practice

26. **Short Response** Find an equation of the line parallel to the graph of $y = -3x + 4$ that passes through $(1, -5)$. *(Lesson 4–6)*

27. **Multiple Choice** In the figure, $m \parallel n$, and *q* is a transversal. If $m\angle 2 = 70$, what is $m\angle 7$? *(Lesson 4–2)*



Ⓐ 55 Ⓑ 70

Ⓒ 110 Ⓓ 140

6–6 The Pythagorean Theorem

What You'll Learn

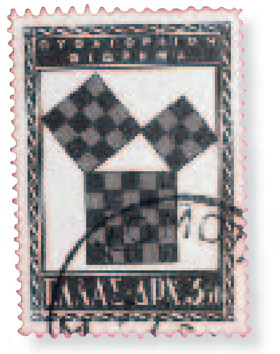
You'll learn to use the Pythagorean Theorem and its converse.

Why It's Important

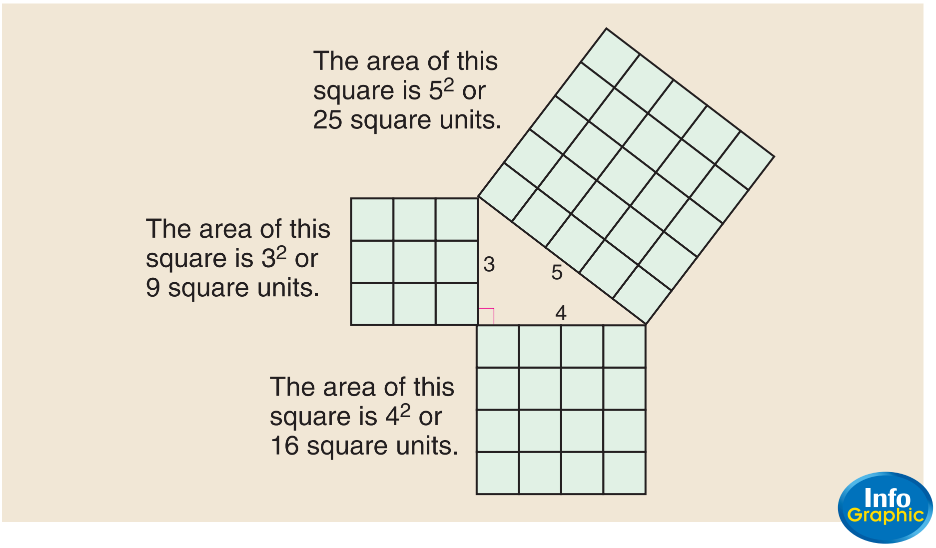
Carpentry

Carpenters use the Pythagorean Theorem to determine the length of roof rafters when they frame a house.
See Example 3.

The stamp shown was issued in 1955 by Greece to honor the 2500th anniversary of the Pythagorean School. Notice the triangle bordered on each side by a checkerboard pattern. Count the number of small squares in each of the three larger squares.



The relationship among 9, 16, and 25 forms the basis for the **Pythagorean Theorem**. It can be illustrated geometrically.



The sides of the right triangle have lengths of 3, 4, and 5 units. The area of the larger square is equal to the total area of the two smaller squares.

$$5^2 = 3^2 + 4^2$$

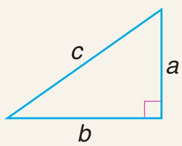
$$25 = 9 + 16$$

This relationship is true for *any* right triangle.

Theorem 6–9 Pythagorean Theorem

Words: In a right triangle, the square of the length of the hypotenuse c is equal to the sum of the squares of the lengths of the legs a and b.

Model:



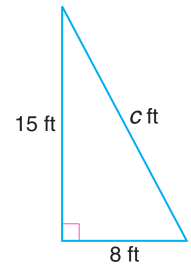
Symbols: $c^2 = a^2 + b^2$

If two measures of the sides of a right triangle are known, the Pythagorean Theorem can be used to find the measure of the third side.

Examples

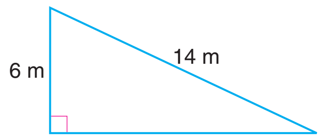
1 **Find the length of the hypotenuse of the right triangle.**

$c^2 = a^2 + b^2$	*Pythagorean Theorem*
$c^2 = 15^2 + 8^2$	*Replace a with 15 and b with 8.*
$c^2 = 225 + 64$	*$15^2 = 225$, $8^2 = 64$*
$c^2 = 289$	*Simplify.*
$c = \sqrt{289}$	*Take the square root of each side.*
2nd [$\sqrt{}$] 289 ENTER 17	
$c = 17$	



The length of the hypotenuse is 17 feet.

2 **Find the length of one leg of a right triangle if the length of the hypotenuse is 14 meters and the length of the other leg is 6 meters.**



$c^2 = a^2 + b^2$	*Pythagorean Theorem*
$14^2 = 6^2 + b^2$	*Replace c with 14 and a with 6.*
$196 = 36 + b^2$	*$14^2 = 196$, $6^2 = 36$*
$196 - 36 = 36 + b^2 - 36$	*Subtract 36 from each side.*
$160 = b^2$	*Simplify.*
$\sqrt{160} = b$	*Take the square root of each side.*
2nd [$\sqrt{}$] 160 ENTER 12.64911064	

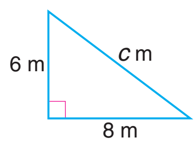
To the nearest tenth, the length of the leg is 12.6 meters.

> **Reading Geometry**
> Always check to be sure that *c* represents the length of the longest side.

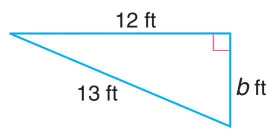
Your Turn

Find the missing measure in each right triangle.

a.



b.



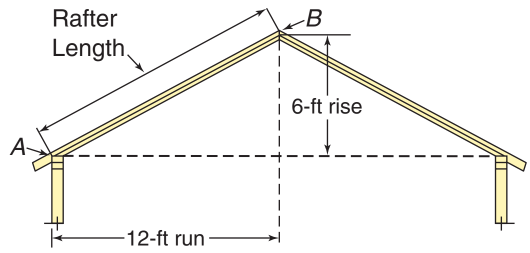
If *c* is the measure of the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.

c. $a = 7$, $b = ?$, $c = 25$

d. $a = ?$, $b = 10$, $c = 20$

Real World
Example 3
Carpentry Link

In pitched roof construction, carpenters build the roof with rafters, one piece at a time. The rise, the run, and the rafter form a right triangle. The rise and run are the legs, and the rafter is the hypotenuse. Find the rafter length for the roof shown at the right. Round to the nearest tenth.



Explore You know the rise is 6 feet and the run is 12 feet. You need to find the length of the rafter.

Plan Let $a = 6$ and $b = 12$. Use the Pythagorean Theorem to find c, the hypotenuse.

Solve

$c^2 = a^2 + b^2$	*Pythagorean Theorem*
$c^2 = 6^2 + 12^2$	*Replace a with 6 and b with 12.*
$c^2 = 36 + 144$	*$6^2 = 36$, $12^2 = 144$*
$c^2 = 180$	*Simplify.*
$c = \sqrt{180}$	*Take the square root of each side.*
$c \approx 13.4$	*Use a calculator.*

The length of the rafter is about 13.4 feet.

Examine Since $10^2 = 100$ and $15^2 = 225$, $\sqrt{180}$ is between 10 and 15. Also, the length of the hypotenuse, 13.4 feet, is longer than the length of either leg.

You can use the converse of the Pythagorean Theorem to test whether a triangle is a right triangle.

Theorem 6–10 Converse of the Pythagorean Theorem	If c is the measure of the longest side of a triangle, a and b are the lengths of the other two sides, and $c^2 = a^2 + b^2$, then the triangle is a right triangle.

CONTENTS

Example 4 **The lengths of the three sides of a triangle are 5, 7, and 9 inches. Determine whether this triangle is a right triangle.**

Since the longest side is 9 inches, use 9 as c, the measure of the hypotenuse.

$c^2 = a^2 + b^2$	*Pythagorean Theorem*
$9^2 \stackrel{?}{=} 5^2 + 7^2$	*Replace c with 9, a with 5, and b with 7.*
$81 \stackrel{?}{=} 25 + 49$	*$9^2 = 81, 5^2 = 25, 7^2 = 49$*
$81 \neq 74$	*Add.*

Since $c^2 \neq a^2 + b^2$, the triangle is *not* a right triangle.

Your Turn

The measures of three sides of a triangle are given. Determine whether each triangle is a right triangle.

e. 20, 21, 28 **f.** 10, 24, 26

Check for Understanding

Communicating Mathematics

Vocabulary
Pythagorean Theorem
Pythagorean triple

1. **State** the Pythagorean Theorem.
2. **Explain** how to find the length of a leg of a right triangle if you know the length of the hypotenuse and the length of the other leg.
3. Writing Math Write a few sentences explaining how you know whether a triangle is a right triangle if you know the lengths of the three sides.

Guided Practice

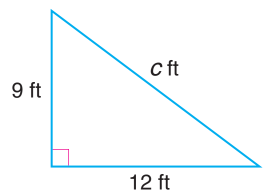
Getting Ready **Find each square root. Round to the nearest tenth, if necessary.**

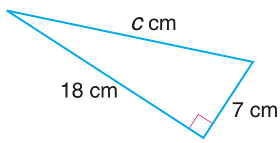
Sample 1: $\sqrt{25}$ **Solution:** 2nd [$\sqrt{\ }$] 25 ENTER 5

Sample 2: $\sqrt{32}$ **Solution:** 2nd [$\sqrt{\ }$] 32 ENTER 5.656854249 $\approx$ 5.7

4. $\sqrt{64}$ 5. $\sqrt{54}$ 6. $\sqrt{126}$ 7. $\sqrt{121}$ 8. $\sqrt{196}$ 9. $\sqrt{87}$

Example 1 **Find the missing measure in each right triangle. Round to the nearest tenth, if necessary.**

10.


11.


Example 2 **If c is the measure of the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.**

12. $a = 30, c = 34, b = ?$ 13. $a = 7, b = 4, c = ?$

Example 4 **The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.**

14. 9 mm, 40 mm, 41 mm **15.** 9 ft, 16 ft, 20 ft

Example 3 **16.** Find the length of the diagonal of a rectangle whose length is 8 meters and whose width is 5 meters.

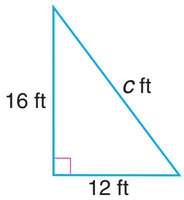
Exercises

Practice

Homework Help	
For Exercises	See Examples
17-28, 36-38	1–3
29-35, 40	4
Extra Practice	
See page 737.	

Find the missing measure in each right triangle. Round to the nearest tenth, if necessary.

17.



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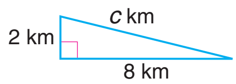

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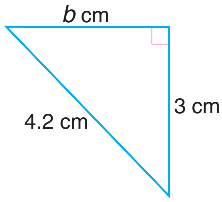

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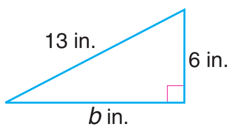

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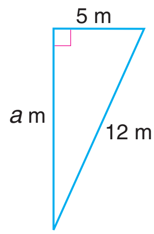
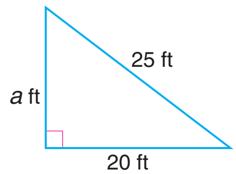


22.



If *c* is the measure of the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.

23. $a = 6, b = 3, c = ?$ **24.** $b = 10, c = 11, a = ?$

25. $c = 29, a = 20, b = ?$ **26.** $a = \sqrt{5}, c = \sqrt{30}, b = ?$

27. $a = \sqrt{7}, b = \sqrt{9}, c = ?$ **28.** $a = \sqrt{11}, c = \sqrt{47}, b = ?$

The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.

29. 11 in., 12 in., 16 in. **30.** 11 cm, 60 cm, 61 cm

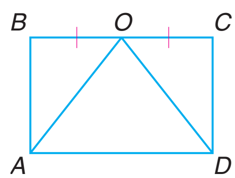
31. 6 ft, 8 ft, 9 ft **32.** 6 mi, 7 mi, 12 mi

33. 45 m, 60 m, 75 m **34.** 1 mm, 1 mm, $\sqrt{2}$ mm

35. Is a triangle with measures 30, 40, and 50 a right triangle? Explain.

36. Find the length of the hypotenuse of a right triangle if the lengths of the legs are 6 miles and 11 miles. Round to the nearest tenth if necessary.

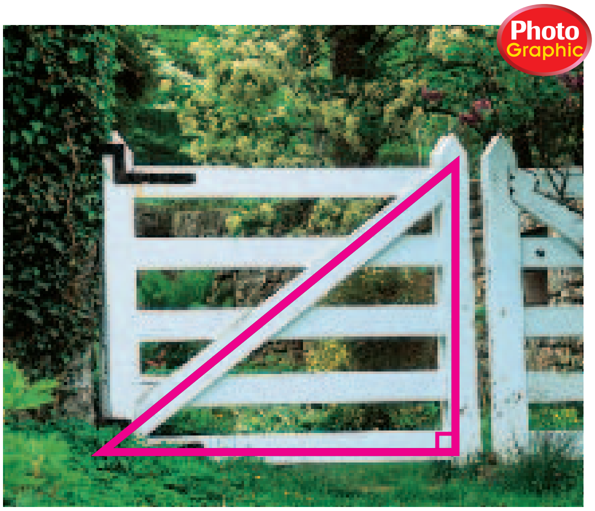
37. Find the measure of the perimeter of rectangle *ABCD* if $OB = OC$, $AO = 40$, and $OB = 32$.



Exercise 37

Applications and Problem Solving

38. **Entertainment** Television sets are measured by the diagonal length of the screen. A 25-inch TV set has a diagonal that measures 25 inches. If the height of the screen is 15 inches, how wide is the screen?



39. **Carpentry** Find the length of a diagonal brace for a rectangular gate that is 5 feet by 4 feet. Round to the nearest tenth.

40. **Critical Thinking** A **Pythagorean triple** is a group of three whole numbers that satisfies the equation $a^2 + b^2 = c^2$, where c is the measure of the hypotenuse. Some common Pythagorean triples are listed below.

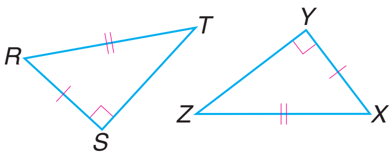
3, 4, 5   9, 12, 15   8, 15, 17   7, 24, 25

a. List three other Pythagorean triples.

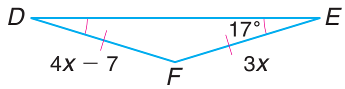
b. Choose any whole number. Then multiply the whole number by each number of one of the Pythagorean triples you listed. Show that the result is also a Pythagorean triple.

Mixed Review

41. Which right angle test for congruence can be used to prove that $\triangle RST \cong \triangle XYZ$? *(Lesson 6–5)*

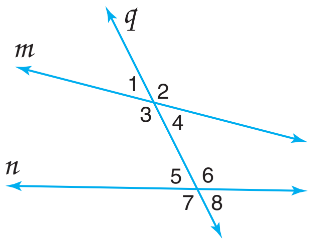


42. **Algebra** In $\triangle DEF$, $\angle D \cong \angle E$ and $m\angle E = 17$. Find $m\angle F$, DF, and FE. *(Lesson 6–4)*



43. Draw an acute scalene triangle. *(Lesson 5–1)*

44. In the figure shown, lines m and n are cut by transversal q. Name two pairs of corresponding angles. *(Lesson 4–3)*



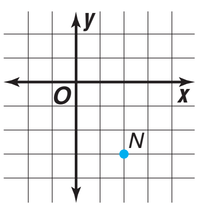
Draw an angle having the given measure. *(Lesson 3–2)*

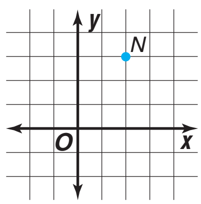
45. 126

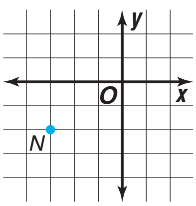
46. 75

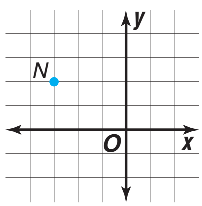
Standardized Test Practice

47. **Multiple Choice** Which shows the graph of $N(2, -3)$? *(Lesson 2–4)*

A


B


C


D


6–7 Distance on the Coordinate Plane

What You'll Learn

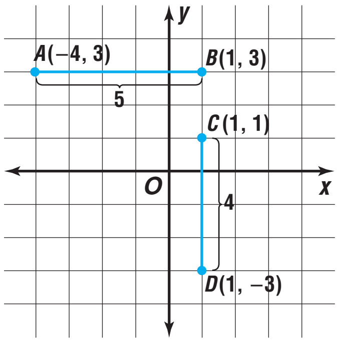
You'll learn to find the distance between two points on the coordinate plane.

Why It's Important

Transportation Knowing how to find the distance between two points can help you determine distance traveled. *See Example 3.*

In Lesson 2–1, you learned how to find the distance between two points on a number line. In this lesson, you will learn how to find the distance between two points on the coordinate plane.

When two points lie on a horizontal line or a vertical line, the distance between the two points can be found by subtracting one of the coordinates. In the coordinate plane below, points A and B lie on a horizontal line, and points C and D lie on a vertical line.



The distance between A and B is $|-4 - 1|$ or 5.

The distance between C and D is $|-3 - 1|$ or 4.

In the following activity, you will learn how to find the distance between two points that do not lie on a horizontal or vertical line.

Look Back

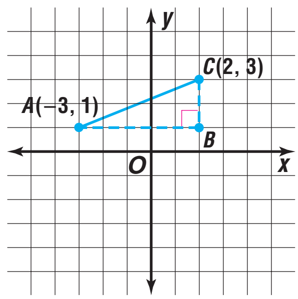
Graphing Ordered Pairs: Lesson 2–4

Hands-On Geometry

Materials: grid paper, straightedge

Step 1 Graph $A(-3, 1)$ and $C(2, 3)$.

Step 2 Draw a horizontal segment from A and a vertical segment from C. Label the intersection B and find the coordinates of B.



Try These

1. What is the measure of the distance between A and B?
2. What is the measure of the distance between B and C?
3. What kind of triangle is $\triangle ABC$?
4. If AB and BC are known, what theorem can be used to find AC?
5. What is the measure of $\overline{AC}$?

In the activity, you found that $(AC)^2 = (AB)^2 + (BC)^2$. By taking the square root of each side of the equation, you find that $AC = \sqrt{(AB)^2 + (BC)^2}$.

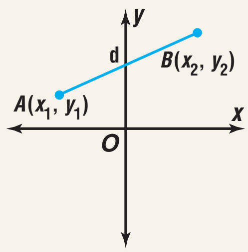
AC = measure of the distance between points A and C
AB = difference of the x-coordinates of A and C
BC = difference of the y-coordinates of A and C

This formula can be generalized to find the distance between any two points.

Theorem 6–11 Distance Formula

Words: If d is the measure of the distance between two points with coordinates (x_1, y_1) and (x_2, y_2), then $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Model:



Example 1

Use the Distance Formula to find the distance between $J(-8, 6)$ and $K(1, -3)$. Round to the nearest tenth, if necessary.

Use the Distance Formula. Replace (x_1, y_1) with (–8, 6) and (x_2, y_2) with $(1, -3)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ *Distance Formula*

$JK = \sqrt{[1 - (-8)]^2 + (-3 - 6)^2}$ *Substitution*

$JK = \sqrt{(9)^2 + (-9)^2}$ *Subtract.*

$JK = \sqrt{81 + 81}$ *$9^2 = 81, (-9)^2 = 81$*

$JK = \sqrt{162}$ *Add.*

$JK \approx 12.7$ *Simplify.*

Your Turn

Find the distance between each pair of points. Round to the nearest tenth, if necessary.

a. $M(0, 3), N(0, 6)$ **b.** $G(-3, 4), H(5, 1)$

You can use the Distance Formula to determine whether a triangle is isosceles given the coordinates of its vertices.

Examples

2 **Determine whether $\triangle ABC$ with vertices $A(-3, 2)$, $B(6, 5)$, and $C(3, -1)$ is isosceles.**

Hint: Draw a picture on a coordinate plane.

An isosceles triangle has at least two congruent sides. Use the Distance Formula to find the measures of the sides of $\triangle ABC$. Then determine if any two are equal.

$$\begin{aligned} AB &= \sqrt{[6-(-3)]^2 + (5-2)^2} \\ &= \sqrt{9^2 + 3^2} \\ &= \sqrt{81 + 9} \\ &= \sqrt{90} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(3-6)^2 + (-1-5)^2} \\ &= \sqrt{(-3)^2 + (-6)^2} \\ &= \sqrt{9 + 36} \\ &= \sqrt{45} \end{aligned}$$

$$\begin{aligned} AC &= \sqrt{[3-(-3)]^2 + (-1-2)^2} \\ &= \sqrt{6^2 + (-3)^2} \\ &= \sqrt{36 + 9} \\ &= \sqrt{45} \end{aligned}$$

$\overline{BC}$ and $\overline{AC}$ have equal measures. Therefore, $\triangle ABC$ is isosceles.

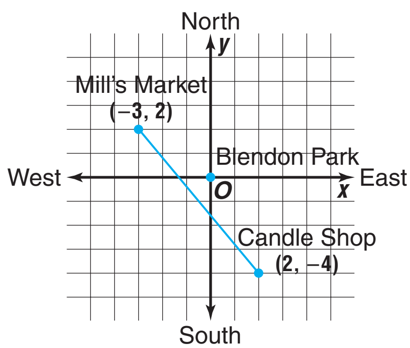
Transportation Link

Real World

3 **Lena takes the bus from Mill's Market to the Candle Shop. Mill's Market is 3 miles west and 2 miles north of Blendon Park. The Candle Shop is 2 miles east and 4 miles south of Blendon Park. How far is the Candle Shop from Mill's Market?**

Let Mill's Market be represented by (x_1, y_1) and the Candle Shop by (x_2, y_2). Then $x_1 = -3$, $y_1 = 2$, $x_2 = 2$, and $y_2 = -4$.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ d &= \sqrt{[2-(-3)]^2 + (-4-2)^2} \\ d &= \sqrt{(5)^2 + (-6)^2} \\ d &= \sqrt{25 + 36} \\ d &= \sqrt{61} \\ d &\approx 7.8 \end{aligned}$$



The Candle Shop is about 7.8 miles from Mill's Market.

Check for Understanding

Communicating Mathematics

1. **State** the Distance Formula for points represented by (x_1, y_1) and (x_2, y_2).

2. **Name** the theorem that is used to determine the Distance Formula in the coordinate plane.

3. **You Decide** Ana says that to find the distance from $A(-3, 2)$ to $B(-7, 5)$, you must evaluate the expression $\sqrt{[-7-(-3)]^2+(5-2)^2}$. Emily disagrees. She says that you must evaluate the expression $\sqrt{[-3-(-7)]^2+(2-5)^2}$. Who is correct? Explain your answer.

Guided Practice

Getting Ready **Find the value of each expression.**

Sample: $(-7+4)^2+[3-(-6)]^2$

Solution: $(-7+4)^2+[3-(-6)]^2 = (-3)^2+[3+6]^2$
$= (-3)^2+9^2$
$= 9+81$ or 90

4. $(6+2)^2+(-5+3)^2$
5. $[-2+(-3)]^2+(2+3)^2$
6. $[-5-(-6)]^2+(4-2)^2$

Example 1

Find the distance between each pair of points. Round to the nearest tenth, if necessary.

7. $E(1, 2), F(3, 4)$
8. $R(-6, 0), S(-2, 0)$
9. $P(5, 6), Q(-3, 1)$

Example 2

10. Determine whether $\triangle FGH$ with vertices $F(-2, 1)$, $G(1, 6)$, and $H(4, 1)$ is isosceles.

11. **Hiking** Tamika and Matthew are going to hike from Cedar Creek Cave to the Ford Nature Center. Cedar Creek Cave is located 3 kilometers west of the ranger's station. The Ford Nature Center is located 2 kilometers east and 4 kilometers north of the ranger's station. **Example 3**
    a. Draw a diagram on a coordinate grid to represent this situation.
    b. What is the distance between Cedar Creek Cave and Ford Nature Center?

Exercises

Practice

Find the distance between each pair of points. Round to the nearest tenth, if necessary.

Homework Help	
For Exercises	See Examples
12–23	1
24–25	2
27–29	3

Extra Practice
See page 738.

12. $A(5, 0), B(12, 0)$
13. $M(2, 3), N(5, 7)$
14. $D(-1, -2), E(-3, -4)$
15. $X(-4, 0), Y(3, -3)$
16. $P(-6, -4), Q(6, -8)$
17. $T(6, 4), U(2, 2)$
18. $B(0, 0), C(-5, 6)$
19. $G(-6, 8), H(-6, -4)$
20. $J(-3, -2), K(3, 1)$
21. $S(-6, -4), T(-3, -7)$

22. Find the distance between $A(-1, 5)$ and $C(3, 5)$.

23. What is the distance between $E(-3, -1)$ and $F(4, -2)$?

24. Is $\triangle MNP$ with vertices $M(1, 4)$, $N(-3, -2)$, and $P(4, -3)$ an isosceles triangle? Explain.

25. Determine whether $\triangle RST$ with vertices $R(1, 5)$, $S(-1, 1)$, and $T(5, 4)$ is scalene. Explain.

26. Triangle FGH has vertices $F(2, 4)$, $G(0, 2)$, and $H(3, -1)$. Determine whether $\triangle FGH$ is a right triangle. Explain.

Applications and Problem Solving

27. Gardening At Memorial Flower Garden, the rose garden is located 25 yards west and 30 yards north of the gazebo. The herb garden is located 35 yards west and 15 yards south of the gazebo.

a. Draw a diagram on a coordinate grid to represent this situation.

b. How far is the herb garden from the rose garden?

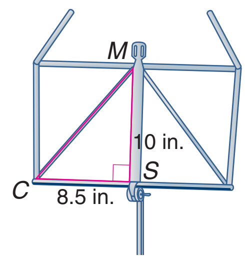
c. What is the distance from the rose garden to the gazebo?

28. Communication To set long-distance rates, telephone companies superimpose an imaginary coordinate plane over the United States. Each ordered pair on this coordinate plane represents the location of a telephone exchange. The phone company calculates the distances between the exchanges in miles to establish long-distance rates. Suppose two exchanges are located at (53, 187) and (129, 71). What is the distance between these exchanges to the nearest mile? The location units are in miles.

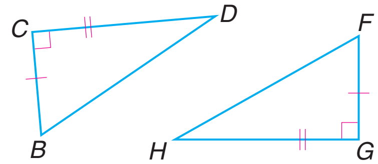
29. Critical Thinking In $\triangle ABC$, the coordinates of the vertices are $A(2, 4)$, $B(-3, 6)$, and $C(-5, -2)$. To the nearest tenth, what is the measure of the median drawn from A to $\overline{BC}$? Include a drawing on a coordinate plane of the triangle and the median.

Mixed Review

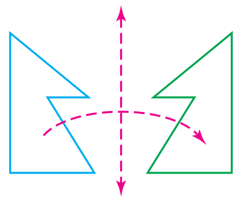
30. Music The frame of the music stand shown contains several triangles. Find the length of the hypotenuse of right triangle MSC if the length of one leg is 10 inches and the length of the other leg is 8.5 inches. Round to the nearest tenth, if necessary. *(Lesson 6–6)*



31. Which right triangle test for congruence can be used to prove that $\triangle BCD \cong \triangle FGH$? *(Lesson 6–5)*

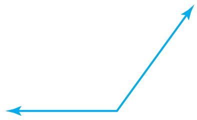


32. Identify the motion shown as a translation, reflection, or rotation. *(Lesson 5–3)*



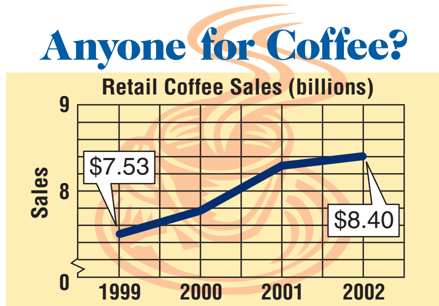
Standardized Test Practice

33. Short Response Classify the angle shown as *acute*, *obtuse*, or *right*. *(Lesson 3–2)*



34. Multiple Choice The line graph shows the retail coffee sales in the United States from 1999 to 2002. Estimate how much more coffee was sold in 2002 than in 2001. *(Statistics Review)*

- (A) $1,000,000
- (B) $10,000,000
- (C) $100,000,000
- (D) $1,000,000,000

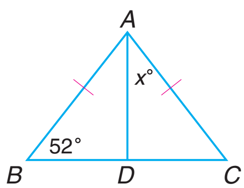


Source: Specialty Coffee Association of America

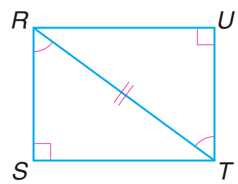
Quiz 2 — Lessons 6–4 through 6–7

1. Find the value of x in $\triangle ABC$ if $AD \perp BC$. *(Lesson 6–4)*

2. Determine whether the pair of right triangles is congruent by LL, HA, LA, or HL. If it is not possible to prove that they are congruent, write *not possible*. *(Lesson 6–5)*



Exercise 1

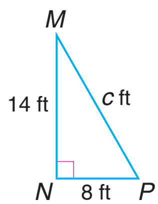


Exercise 2

3. Find c in triangle MNP. Round to the nearest tenth, if necessary. *(Lesson 6–6)*

4. Landscape Tyler is planning to build a triangular garden. The lengths of the sides of the garden are 12 feet, 9 feet, and 15 feet. Will the edges of the garden form a right triangle? *(Lesson 6–6)*

5. Is $\triangle JKL$ with vertices at $J(2, 4)$, $K(-1, -1)$, and $L(5, -1)$ an isosceles triangle? Explain. *(Lesson 6–7)*



Exercise 3

CHAPTER 6 Study Guide and Assessment

Understanding and Using the Vocabulary

After completing this chapter, you should be able to define each term, property, or phrase and give an example or two of each.

interNET CONNECTION Review Activities
For more review activities, visit:
www.geomconcepts.com

altitude *(p. 234)*
angle bisector *(p. 240)*
base *(p. 246)*
centroid *(pp. 230, 244)*
circumcenter *(p. 244)*
concurrent *(p. 230)*
Euler line *(p. 245)*
hypotenuse *(p. 251)*
incenter *(p. 245)*
leg *(pp. 246, 251)*
median *(p. 228)*
nine-point circle *(p. 245)*
orthocenter *(p. 245)*
perpendicular bisector *(p. 235)*
Pythagorean Theorem *(p. 256)*
Pythagorean triple *(p. 261)*

State whether each sentence is *true* or *false*. If false, replace the underlined word(s) to make a true statement.

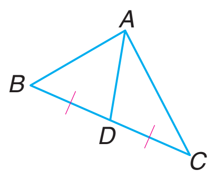


Figure 1

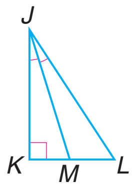


Figure 2

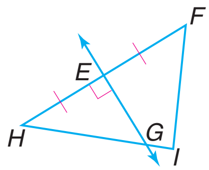


Figure 3

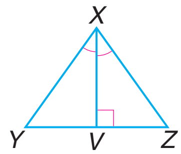


Figure 4

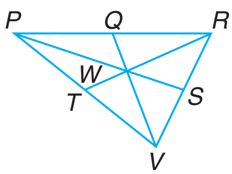
1. In Figure 1, $\overline{AB}$, $\overline{AC}$, and $\overline{AD}$ are <u>concurrent</u>.
2. The point where all of the <u>altitudes</u> of a triangle intersect is called the centroid.
3. In Figure 1, $\overline{AD}$ is a(n) <u>altitude</u> of $\triangle ABC$.
4. In Figure 2, $\overline{JM}$ is a(n) <u>median</u> of $\triangle JKL$.
5. In Figure 2, <u>$(JK)^2 + (JL)^2 = (KL)^2$</u> by the Pythagorean Theorem.
6. In Figure 2, $\overline{JK}$ is a(n) <u>hypotenuse</u> of $\triangle JKL$.
7. In a(n) <u>acute</u> triangle, one of the altitudes lies outside the triangle.
8. In Figure 3, $\overleftrightarrow{EG}$ is a(n) <u>angle bisector</u> of $\overline{HF}$ in $\triangle FHI$.
9. In Figure 4, $\overline{XV}$ is a(n) <u>perpendicular bisector</u> of $\overline{YZ}$.
10. The side opposite the right angle of a right triangle is called the <u>leg</u>.

Skills and Concepts

Objectives and Examples

- **Lesson 6–1** Identify and construct medians in triangles.

In $\triangle PRV$, $\overline{PS}$, $\overline{VQ}$, and $\overline{RT}$ are medians. Find PW if $WS = 7.5$.

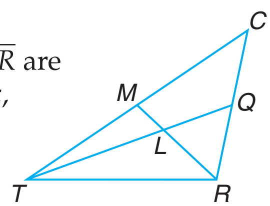


Since $WS = 7.5$,
$PW = 2(7.5)$ or 15.

Review Exercises

Refer to $\triangle PRV$ at the left for Exercises 11–14.

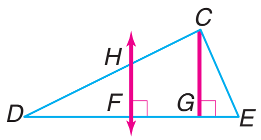
11. Find TW if $WR = 12$.
12. If $PQ = 14.5$, find QR.
13. What is the measure of $\overline{QW}$ if $WV = 11$?
14. If $PV = 20$, find TV.
15. In $\triangle CRT$, $\overline{TQ}$ and $\overline{MR}$ are medians. If $MC = 5x$, $TM = x + 16$, and $CQ = 8x + 6$, find QR.



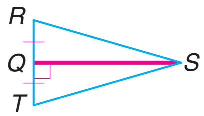
Objectives and Examples	Review Exercises

- **Lesson 6–2** Identify and construct altitudes and perpendicular bisectors in triangles.

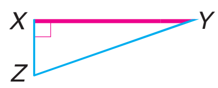
In $\triangle DCE$, $\overline{CG}$ is an altitude, and $\overleftrightarrow{HF}$ is the perpendicular bisector of side DE.



$\overline{SQ}$ is both an altitude and a perpendicular bisector of $\triangle RST$.



For each triangle, tell whether the red segment or line is an *altitude*, a *perpendicular bisector*, *both*, or *neither*.

16.


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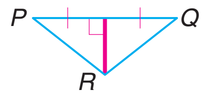

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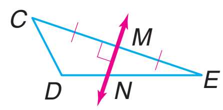

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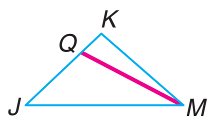
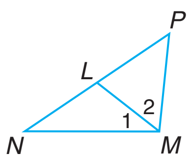


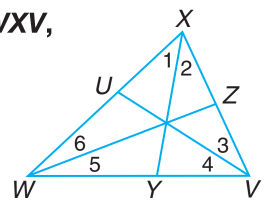
- **Lesson 6–3** Identify and use angle bisectors in triangles.

In $\triangle PMN$, $\overline{ML}$ is an angle bisector of $\angle PMN$. If $m\angle 1 = 55$, find $m\angle PMN$.



$m\angle PMN = 2(m\angle 1)$
$= 2(55)$ or 110

In $\triangle WXV$, $\overline{XY}$ bisects $\angle WXV$, $\overline{UV}$ bisects $\angle XVW$, and $\overline{WZ}$ bisects $\angle XWV$.



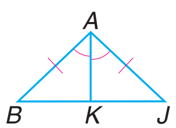
20. If $m\angle 2 = 38$, what is $m\angle 1$?

21. Find $m\angle 3$ if $m\angle WVX = 62$.

22. If $m\angle WXV = 70$ and $m\angle 2 = 3x - 4$, find the value of x.

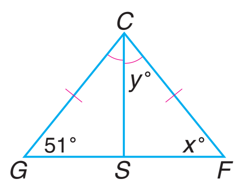
- **Lesson 6–4** Identify and use properties of isosceles triangles.

If $\triangle BAJ$ is isosceles and $\overline{AK}$ bisects $\angle BAJ$, then the following statements are true.



$\angle B \cong \angle J$ $\qquad$ $\overline{AK} \perp \overline{BJ}$ $\qquad$ $\overline{AK}$ bisects $\overline{BJ}$.

Find the values of the variables.

23.


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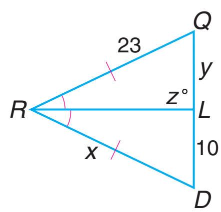
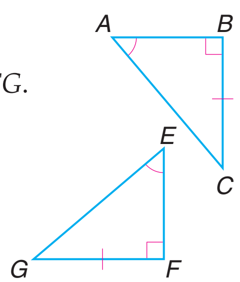


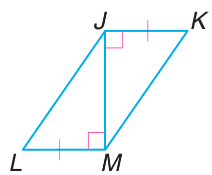
- **Lesson 6–5** Use tests for congruence of right triangles.

Determine if $\triangle ABC \cong \triangle EFG$.
$\angle A \cong \angle E$ and $\overline{BC} \cong \overline{FG}$

By the LA Theorem,
$\triangle ABC \cong \triangle EFG$.



Determine whether each pair of right triangles is congruent by LL, HA, LA, or HL. If it is not possible to prove that they are congruent, write *not possible*.

25.


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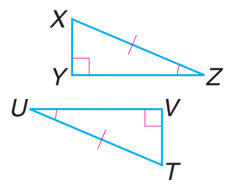
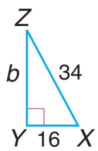


Mixed Problem Solving
See pages 758–765.

Objectives and Examples

- **Lesson 6–6** Use the Pythagorean Theorem and its converse.

Find the value of b in $\triangle XYZ$.



$$a^2 + b^2 = c^2$$
$$16^2 + b^2 = 34^2$$
$$256 + b^2 = 1156$$
$$256 + b^2 - 256 = 1156 - 256$$
$$b^2 = 900$$
$$b = \sqrt{900} \text{ or } 30$$

Review Exercises

If c is the measure of the hypotenuse, find each missing measure. Round to the nearest tenth, if necessary.

27. $a = 16, b = 12, c = ?$

28. $b = 5, c = 14, a = ?$

The lengths of three sides of a triangle are given. Determine whether each triangle is a right triangle.

29. 40 cm, 42 cm, 58 cm

30. 13 ft, 36 ft, 38 ft

- **Lesson 6–7** Find the distance between two points on the coordinate plane.

Use the Distance Formula to find the distance between $A(-3, 7)$ and $B(2, -5)$.

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$AB = \sqrt{[2 - (-3)]^2 + (-5 - 7)^2}$$
$$AB = \sqrt{(5)^2 + (-12)^2}$$
$$AB = \sqrt{25 + 144}$$
$$AB = \sqrt{169} \text{ or } 13$$

Find the distance between each pair of points. Round to the nearest tenth, if necessary.

31. $J(-8, 2), K(0, -4)$

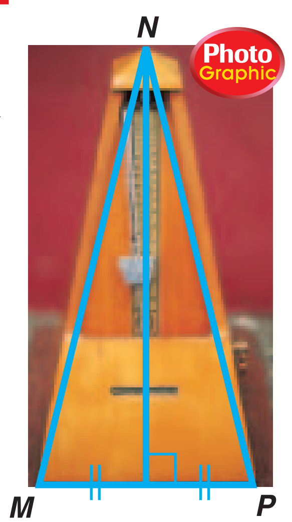
32. $A(3, -7), B(1, -9)$

33. $Y(-5, -2), X(6, -2)$

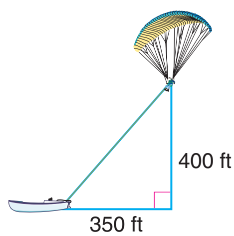
34. Determine whether $\triangle LMN$ with vertices $L(-1, 4)$, $M(5, 1)$, and $N(2, -2)$ is isosceles. Explain.

Applications and Problem Solving

35. Music A *metronome* is a device used to mark exact time using a regularly repeated tick. The body of a metronome resembles an isosceles triangle. In the picture shown at right, is the shaded segment an *altitude*, *perpendicular bisector*, *both*, or *neither* of $\triangle MNP$? *(Lesson 6–4)*



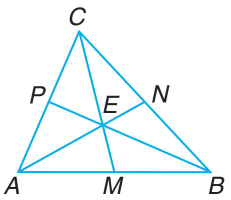
36. Sports Kimiko is parasailing 350 feet away from the boat that pulls her. Suppose she is lifted 400 feet into the air. Find the length of the rope used to keep her attached to the boat. Round to the nearest foot. *(Lesson 6–6)*



CONTENTS

CHAPTER 6 Test

In $\triangle ABC$, $\overline{AN}$, $\overline{BP}$, and $\overline{CM}$ are medians.



Exercises 1–4

1. If $PE = 4$, find EB.

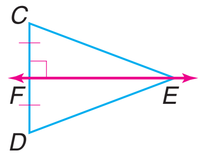
2. Find NB if $CB = 12$.

3. If $AE = 5$, what is EN?

4. If $AM = 2x + 3$, $MB = x + 5$, and $CP = 7x - 6$, find AC.

For each triangle, tell whether the red segment or line is an *altitude*, a *perpendicular bisector*, *both*, or *neither*.

5.



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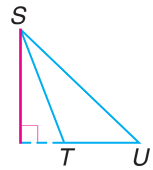

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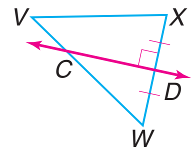
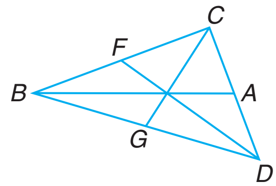


In the figure, $\overline{BA}$ bisects $\angle CBD$, $\overline{CG}$ bisects $\angle BCD$, and $\overline{DF}$ bisects $\angle CDB$.



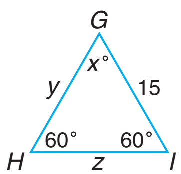
8. Find $m\angle CBD$ if $m\angle ABC = 18$.

9. What is $m\angle BCG$ if $m\angle BCD = 54$?

10. If $m\angle BDF = 3x$ and $m\angle FDC = x + 20$, find x.

For each triangle, find the value of the variables.

11.



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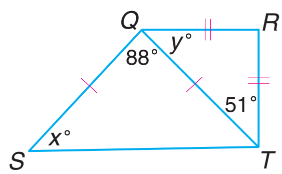

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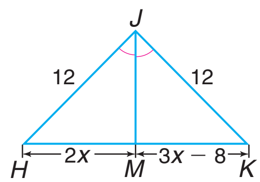
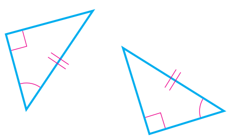


Determine whether each pair of right triangles is congruent by LL, HA, LA, or HL.

14.



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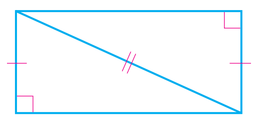

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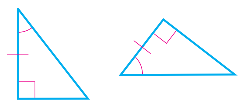
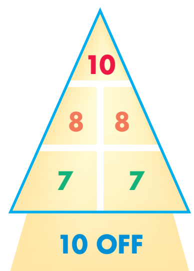


17. Find the length of one leg of a right triangle to the nearest tenth if the length of the hypotenuse is 25 meters and the length of the other leg is 18 meters.

18. The measures of three sides of a triangle are 12, 35, and 37. Determine whether this triangle is a right triangle.

19. What is the distance between $R(-7, 13)$ and $S(1, -2)$?

20. **Games** The scoring area for the game of shuffleboard is an isosceles triangle. Suppose the measure of the vertex angle is 40. What are the measures of the two base angles?



Exercise 20

Algebra Problems

Standardized test problems often ask you to simplify expressions, evaluate expressions, and solve equations.

You may want to review the rules for exponents. For any numbers a and b, and all integers m and n, the following are true.

$$(a^m)^n = a^{mn} \qquad (ab)^m = a^m b^m$$

$$a^m a^n = a^{m+n} \qquad \frac{a^m}{a^n} = a^{m-n}$$

Test-Taking Tip

On multiple-choice problems that ask you to find the value of a variable, you can use a strategy called *working backward*. Replace the variable with each answer choice and see if the statement is true.

Example 1

Evaluate $x^2 - 3x + 4$ if $x = -2$.

(A) 2 (B) 6 (C) 12 (D) 14

Hint Work carefully when combining negative integers.

Solution Replace x with -2. Then perform the operations. Remember the rules for operations using negative numbers.

$$\begin{aligned} x^2 - 3x + 4 &= (-2)^2 - 3(-2) + 4 \\ &= 4 - (-6) + 4 \\ &= 4 + 6 + 4 \\ &= 14 \end{aligned}$$

The answer is D.

Example 2

For which of the following values of x is $\frac{x^2}{x^3}$ the LEAST?

(A) 1 (B) -1 (C) -2
(D) -3 (E) -4

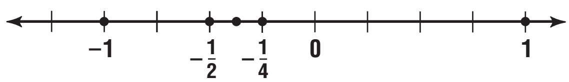
Hint Make problems as simple as possible by simplifying expressions.

Solution Notice that the expression with exponents can be simplified. Start by simplifying it.

$$\frac{x^2}{x^3} = \frac{\cancel{x^2}^{\,1}}{\cancel{x^3}_{\,x^1}} = \frac{1}{x}$$

Now it is easy to evaluate the expression. The problem asks which value makes the expression the smallest. If you substitute each value for x in the expression, which gives you the least or smallest number?

It's easy to check each value. A is 1, B is -1, C is $-\frac{1}{2}$, D is $-\frac{1}{3}$, and E is $-\frac{1}{4}$. Which of these numbers is the least? Think of a number line.



Since -1 is the least, the answer is B.

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 766–781.

After you work each problem, record your answer on the answer sheet provided or on a sheet of paper.

Multiple Choice

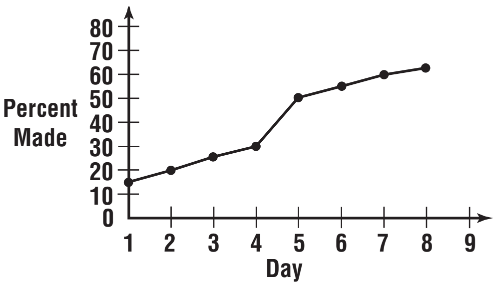
1. Evaluate $x - 3(2) - 4$ if $x = 24$. *(Algebra Review)*

(A) 2 (B) 6 (C) 12 (D) 14

2. Property tax is 2% of the assessed value of a house. How much would the property tax be on a house with an assessed value of $80,000? *(Percent Review)*

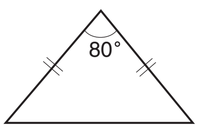
(A) $100 (B) $160 (C) $1000
(D) $1600 (E) $10,000

3. Carl has been practicing basketball free throws. Which statement is best supported by the data shown in the graph? *(Statistics Review)*



(A) By Day 10, Carl should be shooting 80%.
(B) Carl made a total of 60 shots on Day 8.
(C) Carl's performance improved most between Days 7 and 8.
(D) Carl's performance improved most between Days 4 and 5.

4. What is the measure of each base angle of an isosceles triangle in which the vertex angle measures 80°? *(Lesson 5–2)*



(A) 30° (B) 50° (C) 80° (D) 100°

5. The average of x numbers is 16. If the sum of the x numbers is 64, what is the value of x? *(Statistics Review)*

(A) 3 (B) 4 (C) 8
(D) 16 (E) 48

6. The lengths of the sides of a triangle are consecutive even integers. The perimeter of the triangle is 48 inches. What are the lengths of the sides? *(Lesson 1–6)*

(A) 12, 14, 16 (B) 14, 16, 18
(C) 15, 16, 17 (D) 16, 18, 20

7. A box of 36 pens contains 12 blue pens, 14 red pens, and 10 black pens. Three students each successively draw a pen at random from the box and then replace it. If the first two students each draw and then replace a red pen, what is the probability that the third students does *not* draw a red pen? *(Statistics Review)*

(A) $\frac{1}{3}$ (B) $\frac{7}{18}$ (C) $\frac{11}{18}$ (D) $\frac{11}{17}$

8. What is the solution of the inequality $-2 \leq 4 + x$? *(Algebra Review)*

(A) $x \geq -6$ (B) $x \geq 6$
(C) $x \leq -2$ (D) $x \geq 2$

Grid In

9. What is the mean of the ten numbers below? *(Statistics Review)*

$-820, -65, -32, 0, 1, 2, 3, 32, 65, 820$

Short Response

10. Evaluate $x^2 - 1$ for the first eight prime numbers. If you delete the value of $x^2 - 1$ for $x = 2$, what pattern do you see in the other results? (*Hint*: Look at the greatest common factors.) Show your work. Describe the pattern you observed. *(Algebra Review)*