A

Proof

Proof of Theorem 3.1

Using Assumption 3.1, the closed-loop fuzzy system (3.5) can be expressed as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \Big([A_i + B_{2_i} K_j] x(t) + \tilde{B}_{1_i} \tilde{w}(t) \Big) \tag{A.1}$$

where

$$\tilde{B}_{1_i} = \begin{bmatrix} \delta I & I & \delta I & B_{1_i} \end{bmatrix},$$

and the disturbance $\tilde{w}(t)$ is

$$\tilde{w}(t) = \begin{bmatrix} \frac{1}{\delta} F(x(t), t) H_{1_i} x(t) \\ F(x(t), t) H_{2_i} w(t) \\ \frac{1}{\delta} F(x(t), t) H_{3_i} K_j x(t) \\ w(t) \end{bmatrix}. \tag{A.2}$$

Let consider a Lyapunov function

$$V(x(t)) = \gamma x^T(t) Q x(t)$$

where $Q = P^{-1}$. Differentiate $V(x(t))$ along the closed-loop system (A.1) yields

$$\begin{aligned} \dot{V}(x(t)) = \gamma \dot{x}^T(t) Q x(t) + \gamma x^T(t) Q \dot{x}(t) = \\ \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \Big(\gamma x^T(t) (A_i + B_{2_i} K_j)^T Q x(t) + \gamma x^T(t) Q (A_i + B_{2_i} K_j) x(t) \\ + \gamma \tilde{w}^T(t) \tilde{B}_{1_i}^T Q x(t) + \gamma x^T(t) Q \tilde{B}_{1_i} \tilde{w}(t) \Big). \end{aligned} \tag{A.3}$$

Adding and subtracting $-\tilde{z}^T(t)\tilde{z}(t) + \gamma^2 \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{m=1}^{r} \sum_{n=1}^{r} \mu_i \mu_j \mu_m \mu_n \times [\tilde{w}^T(t)\tilde{w}(t)]$ to and from (A.3), we get

$$\dot{V}(x(t)) = \gamma \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n \left[x^T(t)\ \tilde{w}^T(t) \right] \times$$
$$\left(\begin{pmatrix} (A_i + B_{2_i}K_j)^T Q + Q(A_i + B_{2_i}K_j) \\ + \frac{(\tilde{C}_{1_i} + \tilde{D}_{12_i}K_j)^T(\tilde{C}_{1_m} + \tilde{D}_{12_m}K_n)}{\gamma} \end{pmatrix} \ (*)^T \atop \tilde{B}_{1_i}^T Q \qquad\qquad -\gamma I \right) \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}$$
$$-\tilde{z}^T(t)\tilde{z}(t) + \gamma^2 \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)] \quad \text{(A.4)}$$

where

$$\tilde{z}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j[\tilde{C}_{1_i} + \tilde{D}_{12_i}K_j]x(t) \quad \text{(A.5)}$$

with $\tilde{C}_{1_i} = \left[\frac{\gamma\rho}{\delta} H_{1_i}^T\ 0\ \sqrt{2}\lambda\rho H_{4_i}^T\ \sqrt{2}\lambda C_{1_i}^T \right]^T$ and $\tilde{D}_{12_i} = \left[0\ \frac{\gamma\rho}{\delta} H_{3_i}^T\ \sqrt{2}\lambda\rho H_{6_i}^T\ \sqrt{2}\lambda D_{12_i}^T \right]^T$. Pre and post multiply (3.7)-(3.8) by $\begin{pmatrix} Q & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$ yields

$$\begin{pmatrix} (A_i + B_{2_i}K_i)^T Q + Q(A_i + B_{2_i}K_i) & (*)^T & (*)^T \\ \tilde{B}_{1_i}^T Q & -\gamma I & (*)^T \\ \tilde{C}_{1_i} + \tilde{D}_{12_i}K_i & 0 & -\gamma I \end{pmatrix} < 0, \quad \text{(A.6)}$$

$i = 1, 2, \cdots, r$, and

$$\left\{ \begin{pmatrix} (A_i + B_{2_i}K_j)^T Q + Q(A_i + B_{2_i}K_j) & (*)^T & (*)^T \\ \tilde{B}_{1_i}^T Q & -\gamma I & (*)^T \\ \tilde{C}_{1_i} + \tilde{D}_{12_i}K_j & 0 & -\gamma I \end{pmatrix} \right.$$
$$\left. + \begin{pmatrix} (A_j + B_{2_j}K_i)^T Q + Q(A_j + B_{2_j}K_i) & (*)^T & (*)^T \\ \tilde{B}_{1_j}^T Q & -\gamma I & (*)^T \\ \tilde{C}_{1_j} + \tilde{D}_{12_j}K_i & 0 & -\gamma I \end{pmatrix} \right\} < 0, \quad \text{(A.7)}$$

$i < j \leq r$, respectively. Applying the Schur complement on (A.6)-(A.7) and rearranging them, then we have

$$\left(\begin{pmatrix} (A_i + B_{2_i}K_i)^T Q + Q(A_i + B_{2_i}K_i) \\ + \frac{(\tilde{C}_{1_i} + \tilde{D}_{12_i}K_i)^T(\tilde{C}_{1_i} + \tilde{D}_{12_i}K_i)}{\gamma} \end{pmatrix} \ (*)^T \atop \tilde{B}_{1_i}^T Q \qquad\qquad -\gamma I \right) < 0, \quad \text{(A.8)}$$

$i = 1, 2, \cdots, r$, and

$$\left\{\begin{pmatrix} \begin{pmatrix} (A_i + B_{2_i}K_j)^TQ + Q(A_i + B_{2_i}K_j) \\ + \frac{(\tilde{C}_{1_i}+\tilde{D}_{12_i}K_j)^T(\tilde{C}_{1_i}+\tilde{D}_{12_i}K_j)}{\gamma} \end{pmatrix} & (*)^T \\ \tilde{B}_{1_i}^TQ & -\gamma I \end{pmatrix} \right.$$

$$\left. + \begin{pmatrix} \begin{pmatrix} (A_j + B_{2_j}K_i)^TQ + Q(A_j + B_{2_j}K_i) \\ + \frac{(\tilde{C}_{1_j}+\tilde{D}_{12_j}K_i)^T(\tilde{C}_{1_j}+\tilde{D}_{12_j}K_i)}{\gamma} \end{pmatrix} & (*)^T \\ \tilde{B}_{1_j}^TQ & -\gamma I \end{pmatrix}\right\} < 0, \qquad (A.9)$$

$i < j \leq r$, respectively. Using (A.8)-(A.9) and the fact that

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n M_{ij}^TN_{mn} \leq \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j[M_{ij}^TM_{ij} + N_{ij}N_{ij}^T], \qquad (A.10)$$

it is obvious that we have

$$\begin{pmatrix} \begin{pmatrix} (A_i + B_{2_i}K_j)^TQ + Q(A_i + B_{2_i}K_j) \\ + \frac{(\tilde{C}_{1_i}+\tilde{D}_{12_i}K_j)^T(\tilde{C}_{1_i}+\tilde{D}_{12_i}K_j)}{\gamma} \end{pmatrix} & (*)^T \\ \tilde{B}_{1_i}^TQ & -\gamma I \end{pmatrix} < 0 \qquad (A.11)$$

where $i, j = 1, 2, \cdots, r$. Since (A.11) is less than zero and the fact that $\mu_i \geq 0$ and $\sum_{i=1}^{r}\mu_i = 1$, then (A.4) becomes

$$\dot{V}(x(t)) \leq -\tilde{z}^T(t)\tilde{z}(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)]. \qquad (A.12)$$

Integrate both sides of (A.12) yields

$$\int_0^{T_f}\dot{V}(x(t))dt \leq \int_0^{T_f}\Big[-\tilde{z}^T(t)\tilde{z}(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n \times \tilde{w}^T(t)\tilde{w}(t)\Big]dt$$

$$V(x(T_f)) - V(x(0)) \leq \int_0^{T_f}\Big[-\tilde{z}^T(t)\tilde{z}(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n \times \tilde{w}^T(t)\tilde{w}(t)\Big]dt.$$

Using the fact that $x(0) = 0$ and $V(x(T_f)) \geq 0$ for all $T_f \neq 0$, we get

$$\int_0^{T_f}\tilde{z}^T(t)\tilde{z}(t)dt \leq \gamma^2\left[\int_0^{T_f}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)]dt\right]. \quad (A.13)$$

Putting $\tilde{z}(t)$ and $\tilde{w}(t)$ respectively given in (A.5) and (A.2) into (A.13) and using the fact that $\|F(x(t),t)\| \leq \rho$, $\lambda^2 = \Big(1+\rho^2 \sum_{i=1}^r \sum_{j=1}^r [\|H_{2_i}^T H_{2_j}\|]\Big)$ and (A.10), we have

$$\int_0^{T_f} \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Big(2\lambda^2 x^T(t)[C_{1_i} + D_{12_i} K_j]^T [C_{1_i} + D_{12_i} K_j] x(t)$$
$$+2\lambda^2 \rho^2 x^T(t)[H_{4_i} + H_{6_i} K_j]^T [H_{4_i} + H_{6_i} K_j] x(t)\Big)\, dt$$
$$\leq \gamma^2 \lambda^2 \left[\int_0^{T_f} w^T(t) w(t)\, dt\right]. \tag{A.14}$$

Adding and subtracting

$$\lambda^2 z^T(t) z(t) = \lambda^2 \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Big(x^T(t)\big[C_{1_i} + F(x(t),t) H_{4_i} + D_{12_i} K_j$$
$$+F(x(t),t) H_{6_i} K_j\big]^T \big[C_{1_i} + F(x(t),t) H_{4_i} + D_{12_i} K_j$$
$$F(x(t),t) H_{6_i} K_j\big] x(t)\Big)$$

to and from (A.14), one obtains

$$\int_0^{T_f} \Big\{\lambda^2 z^T(t) z(t) + \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Big(2\lambda^2 x^T(t)[C_{1_i} + D_{12_i} K_j]^T \times$$
$$[C_{1_i} + D_{12_i} K_j] x(t) + 2\lambda^2 \rho^2 x^T(t)[H_{4_i} + H_{6_i} K_j]^T [H_{4_i} + H_{6_i} K_j] x(t)$$
$$-\lambda^2 x^T(t)[C_{1_i} + F(x(t),t) H_{4_i}$$
$$+D_{12_i} K_j + F(x(t),t) H_{6_i} K_j]^T [C_{1_i} + F(x(t),t) H_{4_i} + D_{12_i} K_j$$
$$+F(x(t),t) H_{6_i} K_j] x(t)\Big)\Big\}\, dt \leq \gamma^2 \lambda^2 \left[\int_0^{T_f} w^T(t) w(t)\, dt\right]. \tag{A.15}$$

Using the triangular inequality and the fact that $\|F(x(t),t)\| \leq \rho$, we have

$$\lambda^2 \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Big(x^T(t)\, [C_{1_i} + F(x(t),t) H_{4_i} + D_{12_i} K_j + F(x(t),t) H_{6_i} K_j]^T$$
$$\times [C_{1_i} + F(x(t),t) H_{4_i} + D_{12_i} K_j + F(x(t),t) H_{6_i} K_j]\, x(t)\Big)$$
$$\leq \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Big(2\lambda^2 x^T(t)\, [C_{1_i} + D_{12_i} K_j]^T\, [C_{1_i} + D_{12_i} K_j]\, x(t)$$
$$+2\lambda^2 \rho^2 x^T(t)\, [H_{4_i} + H_{6_i} K_j]^T\, [H_{4_i} + H_{6_i} K_j]\, x(t)\Big). \tag{A.16}$$

Using (A.16) on (A.15), we obtain

$$\int_0^{T_f} z^T(t)z(t) \leq \gamma^2 \int_0^{T_f} w^T(t)w(t)\, dt. \tag{A.17}$$

Hence, the inequality (3.3) holds. ■

Proof of Lemma 3.1

The state space form of the fuzzy system model (3.1) with the controller (3.13) is given by

$$\begin{aligned} \dot{\breve{x}}(t) &= \textstyle\sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \left(A_{cl}^{ij}\breve{x}(t) + B_{cl}^{ij}\tilde{w}(t) \right) \\ \breve{z}(t) &= \textstyle\sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j C_{cl}^{ij}\breve{x}(t) \end{aligned} \tag{A.18}$$

where $\breve{x}(t) = \begin{bmatrix} x^T(t) & \hat{x}^T(t) \end{bmatrix}^T$ and the matrix functions A_{cl}^{ij}, B_{cl}^{ij} and C_{cl}^{ij} are defined in Lemma 1 and the disturbance is

$$\tilde{w}(t) = \begin{bmatrix} \frac{1}{\delta}F(x(t),t)H_{1_i}x(t) \\ F(x(t),t)H_{2_i}w(t) \\ \frac{1}{\delta}F(x(t),t)H_{3_i}\hat{C}_j\hat{x}(t) \\ \frac{1}{\delta}F(x(t),t)H_{5_i}x(t) \\ w(t) \\ F(x(t),t)H_{7_i}w(t) \end{bmatrix}. \tag{A.19}$$

Let choose a Lyapunov function

$$V(\breve{x}(t)) = \breve{x}^T(t)Q\breve{x}(t), \tag{A.20}$$

where $Q = P^{-1}$. Differentiate $V(\breve{x}(t))$ along the closed-loop system (A.18) yields

$$\begin{aligned} \dot{V}(\breve{x}(t)) &= \dot{\breve{x}}^T(t)Q\breve{x}(t) + \breve{x}^T(t)Q\dot{\breve{x}}(t) \\ &= \sum_{i=1}^r \sum_{j=1}^r \mu_i\mu_j \Big(\breve{x}^T(t)(A_{cl}^{ij})^T Q\breve{x}(t) + \breve{x}^T(t)QA_{cl}^{ij}\breve{x}(t) \\ &\quad + \tilde{w}^T(t)(B_{cl}^{ij})^T Q\breve{x}(t) + \breve{x}^T(t)QB_{cl}^{ij}\tilde{w}(t) \Big). \end{aligned} \tag{A.21}$$

Add and subtract

$$-\breve{z}^T(t)\breve{z}(t) + \gamma^2 \sum_{i=1}^r \sum_{j=1}^r \sum_{m=1}^r \sum_{n=1}^r \mu_i\mu_j\mu_m\mu_n[\tilde{w}(t)^T\tilde{w}(t)]$$

to and from (A.21) yields

$$\dot{V}(\breve{x}(t)) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\left[\,\breve{x}^T(t)\ \tilde{w}^T(t)\,\right]$$
$$\begin{pmatrix} \begin{pmatrix} (A_{cl}^{ij})^T Q + QA_{cl}^{ij} \\ +(C_{cl}^{ij})^T C_{cl}^{mn} \end{pmatrix} & (*)^T \\ QB_{cl}^{ij} & -\gamma^2 I \end{pmatrix} \begin{bmatrix} \breve{x}(t) \\ \tilde{w}(t) \end{bmatrix}$$
$$-\breve{z}^T(t)\breve{z}(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)]. \quad \text{(A.22)}$$

Now suppose there exits a matrix $P > 0$ such that (3.15) holds, i.e.,

$$\begin{pmatrix} A_{cl}^{ij}P + P(A_{cl}^{ij})^T & (*)^T & (*)^T \\ (B_{cl}^{ij})^T & -\gamma^2 I & (*)^T \\ C_{cl}^{ij}P & 0 & -I \end{pmatrix} < 0. \quad \text{(A.23)}$$

Pre and post multiply (A.23) by $\begin{pmatrix} Q & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$ yields

$$\begin{pmatrix} (A_{cl}^{ij})^T Q + QA_{cl}^{ij} & (*)^T & (*)^T \\ (B_{cl}^{ij})^T Q & -\gamma^2 I & (*)^T \\ C_{cl}^{ij} & 0 & -I \end{pmatrix} < 0. \quad \text{(A.24)}$$

The Schur complement of (A.24) is

$$\begin{pmatrix} (A_{cl}^{ij})^T Q + QA_{cl}^{ij} + (C_{cl}^{ij})^T C_{cl}^{ij} & (*)^T \\ (B_{cl}^{ij})^T & -\gamma^2 I \end{pmatrix} < 0. \quad \text{(A.25)}$$

Using (A.25) and the fact in (A.10) together with the fact that $\mu_i \geq 0$ and $\sum_{i=1}^{r}\mu_i = 1$, then (A.22) becomes

$$\dot{V}(\breve{x}(t)) \leq -\breve{z}^T(t)\breve{z}(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)]. \quad \text{(A.26)}$$

Integrate both sides of (A.26) yields

$$\int_0^{T_f}\dot{V}(\breve{x}(t))dt \leq \int_0^{T_f}\Big(-\breve{z}^T(t)\breve{z}(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n \times [\tilde{w}^T(t)\tilde{w}(t)]\Big)dt$$
$$V(\breve{x}(T_f)) - V(\breve{x}(0)) \leq \int_0^{T_f}\Big(-\breve{z}^T(t)\breve{z}(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n \times [\tilde{w}^T(t)\tilde{w}(t)]\Big)dt.$$

Using the fact that $\breve{x}(0) = 0$ and $V(\breve{x}(T_f)) > 0$ for all $T_f \neq 0$, we have

$$\int_0^{T_f} \breve{z}^T(t)\breve{z}(t)dt \leq \gamma^2 \left[\int_0^{T_f} \sum_{i=1}^r \sum_{j=1}^r \sum_{m=1}^r \sum_{n=1}^r \mu_i \mu_j \mu_m \mu_n [\tilde{w}^T(t)\tilde{w}(t)] \right] dt. \quad (A.27)$$

Putting $\breve{z}(t)$ and $\tilde{w}(t)$ respectively given in (A.18) and (A.19) into (A.27) and using the fact that $\|F(x(t),t)\| \leq \rho$, $\lambda^2 = \Big(1 + \rho^2 \sum_{i=1}^r \sum_{j=1}^r \Big[\|H_{2_i}^T H_{2_j}\| + \|H_{7_i}^T H_{7_j}\|\Big]\Big)$ and (A.10), we have

$$\int_0^{T_f} \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Big(2\lambda^2 \breve{x}^T(t)[C_{1_i} \;\; D_{12_i}\hat{C}_j]^T [C_{1_i} \;\; D_{12_i}\hat{C}_j]\breve{x}(t)$$
$$+2\lambda^2\rho^2 \breve{x}^T(t)[H_{4_i} \;\; H_{6_i}\hat{C}_j]^T [H_{4_i} \;\; H_{6_i}\hat{C}_j]\breve{x}(t) \Big) \, dt \leq \gamma^2 \lambda^2 \left[\int_0^{T_f} w^T(t) w(t) \, dt \right]. \quad (A.28)$$

Adding and subtracting

$$\lambda^2 z^T(t)z(t) = \lambda^2 \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Big(\breve{x}^T(t) \Big[C_{1_i} + F(x(t),t)H_{4_i} \quad D_{12_i}\hat{C}_j$$
$$+F(x(t),t)H_{6_i}\hat{C}_j \Big]^T \Big[C_{1_i} + F(x(t),t)H_{4_i} \quad D_{12_i}\hat{C}_j$$
$$+F(x(t),t)H_{6_i}\hat{C}_j \Big] \breve{x}(t) \Big)$$

to and from (A.28), one obtains

$$\int_0^{T_f} \Big\{ \lambda^2 z^T(t)z(t) + \sum_{i=1}^r \sum_{j=1}^r \mu_i \mu_j \Big(2\lambda^2 \breve{x}^T(t)[C_{1_i} \;\; D_{12_i}\hat{C}_j]^T [C_{1_i} \;\; D_{12_i}\hat{C}_j]\breve{x}(t)$$
$$+2\lambda^2\rho^2 \breve{x}^T(t)[H_{4_i} \;\; H_{6_i}\hat{C}_j]^T [H_{4_i} \;\; H_{6_i}\hat{C}_j]\breve{x}(t)$$
$$-\lambda^2 \breve{x}^T(t)[C_{1_i} + F(x(t),t)H_{4_i} \quad D_{12_i}\hat{C}_j + F(x(t),t)H_{6_i}\hat{C}_j]^T \times$$
$$[C_{1_i} + F(x(t),t)H_{4_i} \quad D_{12_i}\hat{C}_j + F(x(t),t)H_{6_i}\hat{C}_j]\breve{x}(t) \Big) \Big\} \, dt$$
$$\leq \gamma^2 \lambda^2 \left[\int_0^{T_f} w^T(t) w(t) \, dt \right]. \quad (A.29)$$

Using the triangular inequality and the fact that $\|F(x(t),t)\| \leq \rho$, we have

$$\lambda^2 \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j \Big(\breve{x}^T(t) \Big[C_{1_i} + F(x(t),t)H_{4_i} \quad D_{12_i}\hat{C}_j + F(x(t),t)H_{6_i}\hat{C}_j \Big]^T \times$$
$$\Big[C_{1_i} + F(x(t),t)H_{4_i} \quad D_{12_i}\hat{C}_j + F(x(t),t)H_{6_i}\hat{C}_j \Big] \breve{x}(t) \Big)$$
$$\leq \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j \Big(2\lambda^2 \breve{x}^T(t) \Big[C_{1_i} \quad D_{12_i}\hat{C}_j \Big]^T \Big[C_{1_i} \quad D_{12_i}\hat{C}_j \Big] \breve{x}(t)$$
$$+ 2\lambda^2\rho^2 \breve{x}^T(t) \Big[H_{4_i} \quad H_{6_i}\hat{C}_j \Big]^T \Big[H_{4_i} \quad H_{6_i}\hat{C}_j \Big] \breve{x}(t) \Big). \tag{A.30}$$

Using (A.30) on (A.29), we obtain

$$\int_0^{T_f} z^T(t)z(t) \leq \gamma^2 \int_0^{T_f} w^T(t)w(t)\, dt. \tag{A.31}$$

Hence, the inequality (3.3) is guaranteed. ■

Proof of Theorem 5.1

The closed-loop state space form of the fuzzy system model (5.1) with the controller (5.6) is given by

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j \Big([A_i(\imath) + B_{2_i}(\imath)K_j(\imath)]x(t) + [\Delta A_i(\imath) + \Delta B_{2_i}(\imath)K_j(\imath)]x(t)$$
$$+ [B_{1_i}(\imath) + \Delta B_{1_i}(\imath)]w(t) \Big), \quad x(0) = 0, \tag{A.32}$$

or in a more compact form

$$\dot{x}(t) = \textstyle\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j \Big([A_i(\imath) + B_{2_i}(\imath)K_j(\imath)]x(t) + \tilde{B}_{1_i}(\imath)\mathcal{R}(\imath)\tilde{w}(t) \Big) \tag{A.33}$$

where

$$\tilde{B}_{1_i}(\imath) = \big[\, I \; I \; I \; B_{1_i}(\imath) \,\big] \tag{A.34}$$

$$\tilde{w}(t) = \mathcal{R}^{-1}(\imath) \begin{bmatrix} F(x(t),\imath,t)H_{1_i}(\imath)x(t) \\ F(x(t),\imath,t)H_{2_i}(\imath)w(t) \\ F(x(t),\imath,t)H_{3_i}(\imath)K_j(\imath)x(t) \\ w(t) \end{bmatrix}. \tag{A.35}$$

Consider a Lyapunov functional candidate as follows:

$$V(x(t),\imath) = \gamma x^T(t)Q(\imath)x(t), \quad \forall\, \imath \in \mathcal{S}. \tag{A.36}$$

Note that $Q(\imath)$ is constant for each $\imath$. For this choice, we have $V(0,\imath_0) = 0$ and $V(x(t),\imath) \to \infty$ only when $\|x(t)\| \to \infty$.

Now let consider the weak infinitesimal operator $\tilde{\Delta}$ of the joint process $\{(x(t), \imath), t \geq 0\}$, which is the stochastic analog of the deterministic derivative. $\{(x(t), \imath), t \geq 0\}$ is a Markov process with infinitesimal operator given by [80],

$$\begin{aligned}
\tilde{\Delta}V(x(t), \imath) &= \gamma \dot{x}^T(t)Q(\imath)x(t) + \gamma x^T(t)Q(\imath)\dot{x}(t) + \gamma x^T(t)\sum_{k=1}^{s}\lambda_{\imath k}Q(k)x(t) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big(\gamma x^T(t)Q(\imath)\left[(A_i(\imath) + B_{2_i}(\imath)K_j(\imath))\right]x(t) \\
&\quad + \gamma x^T(t)\left[A_i(\imath) + B_{2_i}(\imath)K_j(\imath)\right]^T Q(\imath)x(t) \\
&\quad + \gamma x^T(t)Q(\imath)\tilde{B}_{1_i}(\imath)\mathcal{R}(\imath)\tilde{w}(t) \\
&\quad + \gamma \tilde{w}^T(t)\mathcal{R}(\imath)\tilde{B}_{1_i}^T(\imath)Q(\imath)x(t) + \gamma x^T(t)\sum_{k=1}^{s}\lambda_{\imath k}Q(k)x(t)\Big) \quad \text{(A.37)}
\end{aligned}$$

Adding and subtracting

$$-\aleph^2(\imath)z^T(t)z(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\mathcal{R}(\imath)\tilde{w}(t)]$$

to and from (A.37), we get

$$\begin{aligned}
\tilde{\Delta}V(x(t), \imath) &= -\aleph^2(\imath)z^T(t)z(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\mathcal{R}(\imath)\tilde{w}(t)] \\
&\quad + \aleph^2(\imath)z^T(t)z(t) + \gamma\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\begin{bmatrix} x(t) \\ \tilde{w}(t)\end{bmatrix}^T \times \\
&\quad \begin{pmatrix} \begin{pmatrix} \Big[A_i(\imath) + B_{2_i}(\imath)K_j(\imath)\Big]^T Q(\imath) \\ +Q(\imath)\Big[A_i(\imath) + B_{2_i}(\imath)K_j(\imath)\Big] \\ +\sum_{k=1}^{s}\lambda_{\imath k}Q(k) \end{pmatrix} & (*)^T \\ \mathcal{R}(\imath)\tilde{B}_{1_i}^T(\imath)Q(\imath) & -\gamma\mathcal{R}(\imath) \end{pmatrix}\begin{bmatrix} x(t) \\ \tilde{w}(t)\end{bmatrix}. \quad \text{(A.38)}
\end{aligned}$$

Now let us consider the following terms:

$$\begin{aligned}
&\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\mathcal{R}(\imath)\tilde{w}(t)] = \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n \\
&\times \begin{bmatrix} F(x(t), \imath, t)H_{1_i}(\imath)x(t) \\ F(x(t), \imath, t)H_{2_i}(\imath)w(t) \\ F(x(t), \imath, t)H_{3_i}(\imath)K_j(\imath)x(t) \\ w(t) \end{bmatrix}^T \mathcal{R}^{-1}(\imath)\begin{bmatrix} F(x(t), \imath, t)H_{1_m}(\imath)x(t) \\ F(x(t), \imath, t)H_{2_m}(\imath)w(t) \\ F(x(t), \imath, t)H_{3_m}(\imath)K_n(\imath)x(t) \\ w(t) \end{bmatrix} \\
&\leq \frac{\rho^2(\imath)\gamma^2}{\delta(\imath)}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n x^T(t)\Big\{H_{1_i}^T(\imath)H_{1_m}(\imath) \\
&+ K_j^T(\imath)H_{3_i}^T(\imath)H_{3_m}(\imath)K_n(\imath)\Big\}x(t) + \aleph^2(\imath)\gamma^2 w^T(t)w(t) \qquad \text{(A.39)}
\end{aligned}$$

and

$$\begin{aligned}
\aleph^2(\imath)z^T(t)z(t) &= \aleph^2(\imath)\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n x^T(t)\Big[C_{1_i}(\imath)+ \\
&\quad F(x(t),\imath,t)H_{4_i}(\imath)+D_{12_i}(\imath)K_j(\imath)+F(x(t),\imath,t)H_{6_i}(\imath)K_j(\imath)\Big]^T \\
&\quad \times\Big[C_{1_m}(\imath)+F(x(t),\imath,t)H_{4_m}(\imath) \\
&\quad +D_{12_m}(\imath)K_n(\imath)+F(x(t),\imath,t)H_{6_m}(\imath)K_n(\imath)\Big]x(t) \\
&\le 2\aleph^2(\imath)\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n x^T(t)\Big\{ \\
&\quad \left[C_{1_i}(\imath)+D_{12_i}(\imath)K_j(\imath)\right]^T\left[C_{1_m}(\imath)+D_{12_m}(\imath)K_n(\imath)\right]+ \\
&\quad \rho^2(\imath)\left[H_{4_i}(\imath)+H_{6_i}(\imath)K_j(\imath)\right]^T\times \\
&\quad \left[H_{4_m}(\imath)+H_{6_m}(\imath)K_n(\imath)\right]\Big\}x(t)
\end{aligned} \tag{A.40}$$

where $\aleph(\imath)=\Big(1+\rho^2(\imath)\sum_{i=1}^{r}\sum_{j=1}^{r}[\|H_{2_i}^T(\imath)H_{2_j}(\imath)\|]\Big)^{\frac{1}{2}}$. Hence,

$$\begin{aligned}
&\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\mathcal{R}(\imath)\tilde{w}(t)]+\aleph^2(\imath)z^T(t)z(t) \\
&\le\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\Big(x^T(t)\Big[\tilde{C}_{1_i}(\imath)+\tilde{D}_{12_i}(\imath)K_j(\imath)\Big]^T\mathcal{R}^{-1}(\imath)\times \\
&\Big[\tilde{C}_{1_m}(\imath)+\tilde{D}_{12_m}(\imath)K_n(\imath)\Big]x(t)\Big)+\aleph^2(\imath)\gamma^2w^T(t)w(t)
\end{aligned} \tag{A.41}$$

where

$$\begin{aligned}
\tilde{C}_{1_i}(\imath) &= \left[\,\gamma\rho(\imath)H_{1_i}^T(\imath)\ \sqrt{2}\aleph(\imath)\rho(\imath)H_{4_i}^T(\imath)\ 0\ \sqrt{2}\aleph(\imath)C_{1_i}^T(\imath)\,\right]^T \\
\tilde{D}_{12_i}(\imath) &= \left[\,0\ \sqrt{2}\aleph(\imath)\rho(\imath)H_{6_i}^T(\imath)\ \gamma\rho(\imath)H_{3_i}^T(\imath)\ \sqrt{2}\aleph(\imath)D_{12_i}^T(\imath)\,\right]^T.
\end{aligned}$$

Substituting (A.41) into (A.38), we have

$$\begin{aligned}
\tilde{\Delta}V(x(t),\imath) &\le -\aleph^2(\imath)z^T(t)z(t)+\gamma^2\aleph^2(\imath)w^T(t)w(t) \\
&\quad +\gamma\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}^T\varPhi_{ijmn}(\imath)\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}
\end{aligned} \tag{A.42}$$

where

$$\Phi_{ijmn}(\imath) = \begin{pmatrix} \begin{pmatrix} (A_i(\imath) + B_{2_i}(\imath)K_j(\imath))^T Q(\imath) \\ +Q(\imath)\Big[A_i(\imath) + B_{2_i}(\imath)K_j(\imath)\Big] \\ +\frac{1}{\gamma}\Big[\tilde{C}_{1_i}(\imath) + \tilde{D}_{12_i}(\imath)K_j(\imath)\Big]^T \times \\ \mathcal{R}^{-1}(\imath)\Big[\tilde{C}_{1_m}(\imath) + \tilde{D}_{12_m}(\imath)K_n(\imath)\Big] \\ +\sum_{k=1}^{s}\lambda_{\imath k}Q(k) \end{pmatrix} & (*)^T \\ \mathcal{R}(\imath)\tilde{\bar{B}}_{1_i}^T(\imath)Q(\imath) & -\gamma\mathcal{R}(\imath) \end{pmatrix}. \tag{A.43}$$

Using the fact

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n M_{ij}^T(\imath)N_{mn}(\imath) \leq \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j[M_{ij}^T(\imath)M_{ij}(\imath) + N_{ij}(\imath)N_{ij}^T(\imath)],$$

we can rewrite (A.42) as follows:

$$\begin{aligned} \tilde{\Delta}V(x(t),\imath) &\leq -\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t) \\ &\quad +\gamma\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}^T \Phi_{ij}(\imath) \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix} \\ &= -\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t) \\ &\quad +\gamma\sum_{i=1}^{r}\mu_i^2 \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}^T \Phi_{ii}(\imath) \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix} \\ &\quad +\gamma\sum_{i=1}^{r}\sum_{i<j}\mu_i\mu_j \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}^T \Big(\Phi_{ij}(\imath) + \Phi_{ji}(\imath)\Big) \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix} \end{aligned} \tag{A.44}$$

where

$$\Phi_{ij}(\imath) = \begin{pmatrix} \begin{pmatrix} (A_i(\imath) + B_{2_i}(\imath)K_j(\imath))^T Q(\imath) \\ +Q(\imath)(A_i(\imath) + B_{2_i}(\imath)K_j(\imath)) \\ +\frac{1}{\gamma}\Big[\tilde{C}_{1_i}(\imath) + \tilde{D}_{12_i}(\imath)K_j(\imath)\Big]^T \times \\ \mathcal{R}^{-1}(\imath)\Big[\tilde{C}_{1_i}(\imath) + \tilde{D}_{12_i}(\imath)K_j(\imath)\Big] \\ +\sum_{k=1}^{s}\lambda_{\imath k}Q(k) \end{pmatrix} & (*)^T \\ \mathcal{R}(\imath)\tilde{\bar{B}}_{1_i}^T(\imath)Q(\imath) & -\gamma\mathcal{R}(\imath) \end{pmatrix}. \tag{A.45}$$

Pre and post multiplying (A.45) by

$$\Xi(\imath) = \begin{pmatrix} P(\imath) & 0 \\ 0 & I \end{pmatrix},$$

with $P(\imath) = Q^{-1}(\imath)$, we obtain

$$\Xi(\imath)\Phi_{ij}(\imath)\Xi(\imath) = \begin{pmatrix} \begin{pmatrix} P(\imath)A_i^T(\imath) + Y_j^T(\imath)B_{2_i}^T(\imath) \\ +A_i(\imath)P(\imath) + B_{2_i}(\imath)Y_j(\imath) \\ +\frac{1}{\gamma}\left[\tilde{C}_{1_i}(\imath)P(\imath) + \tilde{D}_{12_i}(\imath)Y_j(\imath)\right]^T \mathcal{R}^{-1}(\imath)\times \\ \left[\tilde{C}_{1_i}(\imath)P(\imath) + \tilde{D}_{12_i}(\imath)Y_j(\imath)\right] \\ +\sum_{k=1}^{s}\lambda_{\imath k}P(\imath)P^{-1}(k)P(\imath) \end{pmatrix} & (*)^T \\ \mathcal{R}(\imath)\tilde{B}_{1_i}^T(\imath) & -\gamma\mathcal{R}(\imath) \end{pmatrix}. \tag{A.46}$$

Note that (A.46) is the Schur complement of $\Psi_{ij}(\imath)$ defined in (5.11). Using (5.9)-(5.10), we learn that

$$\Phi_{ii}(\imath) < 0 \tag{A.47}$$

$$\Phi_{ij}(\imath) + \Phi_{ji}(\imath) < 0. \tag{A.48}$$

Following from (A.44), (A.47) and (A.48), we know that

$$\tilde{\Delta}V(x(t),\imath) < -\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t). \tag{A.49}$$

Applying the operator $\mathbf{E}[\int_0^{T_f}(\cdot)dt]$ on both sides of (A.49), we obtain

$$\mathbf{E}\left[\int_0^{T_f}\tilde{\Delta}V(x(t),\imath)dt\right] < \mathbf{E}\left[\int_0^{T_f}(-\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t))dt\right]. \tag{A.50}$$

From the Dynkin's formula [75], it follows that

$$\mathbf{E}\left[\int_0^{T_f}\tilde{\Delta}V(x(t),\imath)dt\right] = \mathbf{E}[V(x(T_f),\imath(T_f))] - \mathbf{E}[V(x(0),\imath(0))]. \tag{A.51}$$

Substitute (A.51) into (A.50) yields

$$\begin{aligned} 0 < \mathbf{E}&\left[\int_0^{T_f}(-\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t))dt\right] \\ &-\mathbf{E}[V(x(T_f),\imath(T_f))] + \mathbf{E}[V(x(0),\imath(0))]. \end{aligned}$$

Using (A.49) and the fact that $V(x(0) = 0, \imath(0)) = 0$ and $V(x(T_f), \imath(T_f)) > 0$, we have

$$\mathbf{E}\left[\int_0^{T_f}\left\{z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right\}dt\right] < 0. \tag{A.52}$$

Hence, the inequality (5.5) holds. This completes the proof of Theorem 6. ■

Proof of Lemma 5.1

The closed-loop state space form of the fuzzy system model (5.1) with the controller (5.24) is given by

$$\begin{aligned} \dot{\breve{x}}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big(A_{cl}^{ij}(\imath)\breve{x}(t) + B_{cl}^{ij}(\imath)\tilde{w}(t)\Big) \\ \breve{z}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j C_{cl}^{ij}(\imath)\breve{x}(t) \end{aligned} \tag{A.53}$$

where $\breve{x}(t) = \begin{bmatrix} x^T(t) & \hat{x}^T(t) \end{bmatrix}^T$ and the matrix functions $A_{cl}^{ij}(\imath)$, $B_{cl}^{ij}(\imath)$ and $C_{cl}^{ij}(\imath)$ are defined in Lemma 3 and the disturbance is

$$\tilde{w}(t) = \begin{bmatrix} \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{1_i}(\imath)x(t) \\ F(x(t),\imath,t)H_{2_i}(\imath)w(t) \\ \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{3_i}(\imath)\hat{C}_j(\imath)\hat{x}(t) \\ \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{5_i}(\imath)x(t) \\ w(t) \\ F(x(t),\imath,t)H_{7_i}(\imath)w(t) \end{bmatrix}.$$

Let choose a stochastic Lyapunov function

$$V(\breve{x}(t),\imath) = \breve{x}^T(t)P(\imath)\breve{x}(t) \;\; \forall\, \imath \in \mathcal{S} \tag{A.54}$$

where $P(\imath)$ is a constant positive definite matrix for each $\imath$. For this choice, we have $V(0,\imath_0) = 0$ and $V(\breve{x}(t),\imath) \to \infty$ only when $\|\breve{x}(t)\| \to \infty$.

Consider the weak infinitesimal operator $\tilde{\Delta}$ of the joint process $\{(\breve{x}(t),\imath), t \geq 0\}$, which is the stochastic analog of the deterministic derivative. $\{(\breve{x}(t),\imath), t \geq 0\}$ is a Markov process with infinitesimal operator given by [80],

$$\begin{aligned} \tilde{\Delta}V(\breve{x}(t),\imath) &= \dot{\breve{x}}^T(t)P(\imath)\breve{x}(t) + \breve{x}^T(t)P(\imath)\dot{\breve{x}}(t) + \breve{x}^T(t)\sum_{k=1}^{s}\lambda_{\imath k}P(k)\breve{x}^T(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big(\breve{x}^T(t)(A_{cl}^{ij}(\imath))^T P(\imath)\breve{x}(t) + \breve{x}^T(t)P(\imath)A_{cl}^{ij}(\imath)\breve{x}(t) \\ &\quad + \tilde{w}^T(t)(B_{cl}^{ij}(\imath))^T P(\imath)\breve{x}(t) + \breve{x}^T(t)P(\imath)B_{cl}^{ij}(\imath)\tilde{w}(t) \\ &\quad + \breve{x}^T(t)\sum_{k=1}^{s}\lambda_{\imath k}P(k)\breve{x}^T(t)\Big). \end{aligned} \tag{A.55}$$

Adding and subtracting

$$-\aleph^2(\imath)z^T(t)z(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)]$$

to and from (A.55), we get

$$\tilde{\Delta}V(x(t),\imath) = -\aleph^2(\imath)z^T(t)z(t) + \gamma^2 \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)]$$
$$+\aleph^2(\imath)z^T(t)z(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n \begin{bmatrix} \breve{x}(t) \\ \tilde{w}(t) \end{bmatrix}^T \times$$
$$\begin{pmatrix} \begin{pmatrix} (A_{cl}^{ij}(\imath))^T P(\imath) + P(\imath)A_{cl}^{ij}(\imath) \\ +\sum_{k=1}^{s} \lambda_{\imath k}P(k) \end{pmatrix} & (*)^T \\ (B_{cl}^{ij}(\imath))^T P(\imath) & -\gamma^2 I \end{pmatrix} \begin{bmatrix} \breve{x}(t) \\ \tilde{w}(t) \end{bmatrix}. \qquad \text{(A.56)}$$

Now let us consider the following terms:

$$\gamma^2 \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)] = \gamma^2 \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n \times$$
$$\begin{bmatrix} \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{1_i}(\imath)x(t) \\ F(x(t),\imath,t)H_{2_i}(\imath)w(t) \\ \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{3_i}(\imath)\hat{C}_j(\imath)\hat{x}(t) \\ \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{5_i}(\imath)x(t) \\ w(t) \\ F(x(t),\imath,t)H_{7_i}(\imath)w(t) \end{bmatrix}^T \begin{bmatrix} \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{1_m}(\imath)x(t) \\ F(x(t),\imath,t)H_{2_m}(\imath)w(t) \\ \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{3_m}(\imath)\hat{C}_n(\imath)\hat{x}(t) \\ \frac{1}{\delta(\imath)}F(x(t),\imath,t)H_{5_m}(\imath)x(t) \\ w(t) \\ F(x(t),\imath,t)H_{7_m}(\imath)w(t) \end{bmatrix}$$
$$\leq \frac{\gamma^2\rho^2(\imath)}{\delta^2(\imath)} \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n \breve{x}^T(t) \times$$
$$\left[\begin{pmatrix} H_{1_i}^T(\imath)H_{1_m}(\imath) \\ +H_{5_i}^T(\imath)H_{5_m}(\imath) \end{pmatrix} \begin{pmatrix} \hat{C}_j^T(\imath)H_{3_i}^T(\imath)\times \\ H_{3_m}(\imath)\hat{C}_n(\imath) \end{pmatrix} \right] \breve{x}(t) + \aleph^2(\imath)\gamma^2 w^T(t)w(t) \qquad \text{(A.57)}$$

and

$$\aleph^2(\imath)z^T(t)z(t) = \aleph^2(\imath) \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n \breve{x}^T(t) \times$$
$$\left[C_{1_i}(\imath) + F(x(t),\imath,t)H_{4_i}(\imath) \quad D_{12_i}(\imath)\hat{C}_j(\imath) + F(x(t),\imath,t)H_{6_i}(\imath)\hat{C}_j(\imath) \right]^T \times$$
$$\left[C_{1_m}(\imath) + F(x(t),\imath,t)H_{4_m}(\imath) \quad D_{12_m}(\imath)\hat{C}_n(\imath) + F(x(t),\imath,t)H_{6_m}(\imath)\hat{C}_n(\imath) \right] \breve{x}(t)$$
$$\leq \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n \Big(2\aleph^2(\imath)\breve{x}^T(t) \left[C_{1_i}(\imath) \quad D_{12_i}(\imath)\hat{C}_j(\imath) \right]^T \times$$
$$\left[C_{1_m}(\imath) \quad D_{12_m}(\imath)\hat{C}_n(\imath) \right] \breve{x}(t) + 2\aleph^2(\imath)\rho^2(\imath)\tilde{x}^T(t) \times$$
$$\left[H_{4_i}(\imath) \quad H_{6_i}(\imath)\hat{C}_j(\imath) \right]^T \left[H_{4_m}(\imath) \quad H_{6_m}(\imath)\hat{C}_n(\imath) \right] \breve{x}(t) \Big) \qquad \text{(A.58)}$$

where $\aleph(\imath) \geq \Big(1 + \rho^2(\imath)\Big[\|H_{2_i}^T(\imath)H_{2_j}(\imath)\| + \|H_{7_i}^T(\imath)H_{7_j}(\imath)\|\Big]\Big)^{\frac{1}{2}}$. Hence,

$$\gamma^2 \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)] + \aleph^2(\imath)z^T(t)z(t)$$
$$\leq \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n\Big(\breve{x}^T(t)\Big[\tilde{C}_{1_i}(\imath) \quad \tilde{D}_{12_i}(\imath)\hat{C}_j(\imath)\Big]^T \times$$
$$\Big[\tilde{C}_{1_m}(\imath) \quad \tilde{D}_{12_m}(\imath)\hat{C}_n(\imath)\Big]\breve{x}(t)\Big) + \aleph^2(\imath)\gamma^2 w^T(t)w(t) \tag{A.59}$$

where

$$\tilde{C}_{1_i}(\imath) = \Big[\ \tfrac{\gamma\rho(\imath)}{\delta(\imath)}H_{1_i}^T(\imath)\ \ 0\ \ \tfrac{\gamma\rho(\imath)}{\delta(\imath)}H_{5_i}^T(\imath)\ \ \sqrt{2}\aleph(\imath)\rho(\imath)H_{4_i}^T(\imath)\ \ \sqrt{2}\aleph(\imath)C_{1_i}^T(\imath)\Big]^T$$
$$\tilde{D}_{12_i}(\imath) = \Big[0\ \ \tfrac{\gamma\rho(\imath)}{\delta(\imath)}H_{3_i}^T(\imath)\ \ 0\ \ \sqrt{2}\aleph(\imath)\rho(\imath)H_{6_i}^T(\imath)\ \ \sqrt{2}\aleph(\imath)D_{12_i}^T(\imath)\Big]^T.$$

Substituting (A.59) into (A.56), we have

$$\tilde{\Delta}V(x(t),\imath) \leq -\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t)$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}^T \Omega_{ijmn}(\imath) \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix} \tag{A.60}$$

where

$$\Omega_{ijmn}(\imath) = \begin{pmatrix} \begin{pmatrix} (A_{cl}^{ij}(\imath))^T P(\imath) + P(\imath)A_{cl}^{ij}(\imath) \\ +(C_{cl}^{ij}(\imath))^T C_{cl}^{mn}(\imath) + \sum_{k=1}^{s}\lambda_{\imath k}P(k) \end{pmatrix} & (*)^T \\ (B_{cl}^{ij}(\imath))^T P(\imath) & -\gamma^2 I \end{pmatrix}. \tag{A.61}$$

Using the fact

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r} \mu_i\mu_j\mu_m\mu_n M_{ij}^T(\imath)N_{mn}(\imath) \leq \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j[M_{ij}^T(\imath)M_{ij}(\imath)$$
$$+N_{ij}(\imath)N_{ij}^T(\imath)],$$

we can rewrite (A.61) as follows:

$$\tilde{\Delta}V(x(t),\imath) \leq -\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t)$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix}^T \Omega_{ij}(\imath) \begin{bmatrix} x(t) \\ \tilde{w}(t) \end{bmatrix} \tag{A.62}$$

where

$$\Omega_{ij}(\imath) = \begin{pmatrix} \begin{pmatrix} (A_{cl}^{ij}(\imath))^T P(\imath) + P(\imath)A_{cl}^{ij}(\imath) \\ +(C_{cl}^{ij}(\imath))^T C_{cl}^{ij}(\imath) + \sum_{k=1}^{s}\lambda_{\imath k}P(k) \end{pmatrix} & (*)^T \\ (B_{cl}^{ij}(\imath))^T P(\imath) & -\gamma^2 I \end{pmatrix}. \tag{A.63}$$

Note that (A.63) is the Schur complement of (5.26). Using the inequality (5.26), we have

$$\tilde{\Delta}V(x(t),\imath) < -\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t). \tag{A.64}$$

Applying the operator $\mathbf{E}[\int_0^{T_f}(\cdot)dt]$ on both sides of (A.64), we obtain

$$\mathbf{E}\left[\int_0^{T_f} \tilde{\Delta}V(x(t),\imath)dt\right] < \mathbf{E}\left[\int_0^{T_f} (-\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t))dt\right]. \tag{A.65}$$

From the Dynkin's formula [75], it follows that

$$\mathbf{E}\left[\int_0^{T_f} \tilde{\Delta}V(x(t),\imath)dt\right] = \mathbf{E}[V(x(T_f),\imath(T_f))] - \mathbf{E}[V(x(0),\imath(0))]. \tag{A.66}$$

Substitute (A.66) into (A.65) yields

$$\begin{aligned} 0 < \mathbf{E}&\left[\int_0^{T_f} (-\aleph^2(\imath)z^T(t)z(t) + \gamma^2\aleph^2(\imath)w^T(t)w(t))dt\right] \\ &-\mathbf{E}[V(x(T_f),\imath(T_f))] + \mathbf{E}[V(x(0),\imath(0))]. \end{aligned}$$

Using (A.49) and the fact that $V(x(0) = 0, \imath(0)) = 0$ and $V(x(T_f),\imath(T_f)) > 0$, we have

$$\mathbf{E}\left[\int_0^{T_f} \left\{z^T(t)z(t) - \gamma^2 w^T(t)w(t)\right\}dt\right] < 0. \tag{A.67}$$

Hence the inequality (5.5) holds. This completes the proof of Lemma 3. ■

Proof of Theorem 8.2

Suppose the inequalities (8.36)-(8.38) hold, then the matrices X_0 and Y_0 are of the following forms:

$$X_0 = \begin{pmatrix} X_1 & X_2 \\ 0 & X_3 \end{pmatrix} \text{ and } Y_0 = \begin{pmatrix} Y_1 & Y_2 \\ 0 & Y_3 \end{pmatrix}$$

with $X_1 = X_1^T > 0$, $X_3 = X_3^T > 0$, $Y_1 = Y_1^T > 0$ and $Y_3 = Y_3^T > 0$. Substituting X_0 and Y_0 into (8.47), respectively, we have

$$X_\varepsilon = \left\{X_0 + \varepsilon\tilde{X}\right\}E_\varepsilon = \begin{pmatrix} X_1 & \varepsilon X_2 \\ \varepsilon X_2^T & \varepsilon X_3 \end{pmatrix}. \tag{A.68}$$

and

$$Y_\varepsilon^{-1} = \left\{Y_0^{-1} + \varepsilon N_\varepsilon\right\}E_\varepsilon = \begin{pmatrix} Y_1^{-1} & -\varepsilon Y^{-1}Y_2Y_3^{-1} \\ -\varepsilon(Y^{-1}Y_2Y_3^{-1})^T & \varepsilon Y_3^{-1} \end{pmatrix}. \tag{A.69}$$

Clearly, $X_\varepsilon = X_\varepsilon^T$, and $Y_\varepsilon^{-1} = (Y_\varepsilon^{-1})^T$. Knowing the fact that the inverse of a symmetric matrix is a symmetric matrix, we learn that Y_ε is a symmetric matrix. Using the matrix inversion lemma, we can see that

$$Y_\varepsilon = E_\varepsilon^{-1}\left\{Y_0 + \varepsilon\tilde{Y}\right\} \tag{A.70}$$

where $\tilde{Y} = Y_0 N_\varepsilon (I + \varepsilon Y_0 N_\varepsilon)^{-1} Y_0$. Employing the Schur complement, one can show that there exists a sufficiently small $\hat{\varepsilon}$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, (8.26) and (8.27) hold.

Now, we need to show that

$$\begin{pmatrix} X_\varepsilon & I \\ I & Y_\varepsilon \end{pmatrix} > 0. \tag{A.71}$$

By the Schur complement, it is equivalent to showing that

$$X_\varepsilon - Y_\varepsilon^{-1} > 0. \tag{A.72}$$

Substituting (A.68) and (A.69) into the left hand side of (A.72), we get

$$\begin{bmatrix} X_1 - Y_1^{-1} & \varepsilon(X_2 + Y_1^{-1}Y_2Y_3^{-1}) \\ \varepsilon(X_2 + Y_1^{-1}Y_2Y_3^{-1})^T & \varepsilon(X_3 - Y_3^{-1}) \end{bmatrix}. \tag{A.73}$$

The Schur complement of (8.36) is

$$\begin{bmatrix} X_1 - Y_1^{-1} & 0 \\ 0 & X_3 - Y_3^{-1} \end{bmatrix} > 0. \tag{A.74}$$

According to (A.74), we learn that

$$X_1 - Y_1^{-1} > 0 \qquad \text{and} \qquad X_3 - Y_3^{-1} > 0. \tag{A.75}$$

Using (A.75) and the Schur complement, it can be shown that there exists a sufficiently small $\hat{\varepsilon} > 0$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, (8.25) holds.

Next, employing (A.68), (A.69) and (A.70), the controller's matrices given in (8.34) can be re-expressed as follows:

$$\begin{aligned} \mathcal{B}_i(\varepsilon) &= \left[Y_0^{-1} - X_0\right]\hat{B}_i + \varepsilon\left[N_\varepsilon - \tilde{X}\right]\hat{B}_i \triangleq \mathcal{B}_{0_i} + \varepsilon\mathcal{B}_{\varepsilon_i} \\ \mathcal{C}_i(\varepsilon) &= \hat{C}_i Y_0^T + \varepsilon\hat{C}_i\tilde{Y}^T \triangleq \mathcal{C}_{0_i} + \varepsilon\mathcal{C}_{\varepsilon_i}. \end{aligned} \tag{A.76}$$

Substituting (A.68), (A.69), (A.70) and (A.76) into (8.32) and (8.33), and pre-post multiplying (8.32) by $\begin{pmatrix} E_\varepsilon & 0 \\ 0 & I \end{pmatrix}$, we, respectively, obtain

$$\Psi_{11_{ij}} + \psi_{11_{ij}} \;\text{ and }\; \Psi_{22_{ij}} + \psi_{22_{ij}} \tag{A.77}$$

where the ε-independent linear matrices $\Psi_{11_{ij}}$ and $\Psi_{22_{ij}}$ are defined in (8.43) and (8.44), respectively and the ε-dependent linear matrices are

$$\psi_{11_{ij}} = \varepsilon \begin{pmatrix} A_i\tilde{Y}^T + \tilde{Y}A_i^T + B_{2_i}\mathcal{C}_{\varepsilon_j} + \mathcal{C}_{\varepsilon_i}^T B_{2_j}^T & (*)^T \\ \left[\tilde{Y}\tilde{C}_{1_i}^T + \mathcal{C}_{\varepsilon_i}^T \tilde{D}_{12_j}^T\right]^T & 0 \end{pmatrix} \tag{A.78}$$

$$\psi_{22_{ij}} = \varepsilon \begin{pmatrix} A_i^T\tilde{X} + \tilde{X}^T A_i + \mathcal{B}_{\varepsilon_i} C_{2_j} + C_{2_i}^T \mathcal{B}_{\varepsilon_j}^T & (*)^T \\ \left[\tilde{X}\tilde{B}_{1_i} + \mathcal{B}_{\varepsilon_i}\tilde{D}_{21_j}\right]^T & 0 \end{pmatrix}. \tag{A.79}$$

Note that the ε-dependent linear matrices tend to zero when ε approaches zero.

Employing (8.39)–(8.42) and knowing the fact that for any given negative definite matrix $\mathcal{W}$, there exists an $\varepsilon > 0$ such that $\mathcal{W} + \varepsilon I < 0$, one can show that there exists a sufficiently small $\hat{\varepsilon} > 0$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, (8.28)–(8.31) hold. Since (8.25)-(8.31) hold, using Lemma 8.2, the inequality (3.3) holds. ■

Proof of Theorem 10.2

Suppose the inequalities (10.56)-(10.58) hold, then the matrices $X_0(\imath)$ and $Y_0(\imath)$ are of the following forms:

$$X_0(\imath) = \begin{pmatrix} X_1(\imath) & X_2(\imath) \\ 0 & X_3(\imath) \end{pmatrix} \text{ and } Y_0(\imath) = \begin{pmatrix} Y_1(\imath) & Y_2(\imath) \\ 0 & Y_3(\imath) \end{pmatrix}$$

with $X_1(\imath) = X_1^T(\imath) > 0$, $X_3(\imath) = X_3^T(\imath) > 0$, $Y_1(\imath) = Y_1^T(\imath) > 0$ and $Y_3(\imath) = Y_3^T(\imath) > 0$. Substituting $X_0(\imath)$ and $Y_0(\imath)$ into (10.67)-(10.68), respectively, we have

$$X_\varepsilon(\imath) = \begin{pmatrix} X_1(\imath) & \varepsilon X_2(\imath) \\ \varepsilon X_2^T(\imath) & \varepsilon X_3(\imath) \end{pmatrix} \tag{A.80}$$

and

$$Y_\varepsilon^{-1}(\imath) = \begin{pmatrix} Y_1^{-1}(\imath) & -\varepsilon Y^{-1}(\imath)Y_2(\imath)Y_3^{-1}(\imath) \\ -\varepsilon\left(Y^{-1}(\imath)Y_2(\imath)Y_3^{-1}(\imath)\right)^T & \varepsilon Y_3^{-1}(\imath) \end{pmatrix}. \tag{A.81}$$

Clearly, $X_\varepsilon(\imath) = X_\varepsilon^T(\imath)$, and $Y_\varepsilon^{-1}(\imath) = (Y_\varepsilon^{-1}(\imath))^T$. Knowing the fact that the inverse of a symmetric matrix is a symmetric matrix, we learn that $Y_\varepsilon(\imath)$ is a symmetric matrix. Using the matrix inversion lemma, we can see that $Y_\varepsilon(\imath) = E_\varepsilon^{-1}\left\{Y_0(\imath) + \varepsilon\tilde{Y}(\imath)\right\}$ where $\tilde{Y}(\imath) = Y_0(\imath)N_\varepsilon(\imath)(I + \varepsilon Y_0(\imath)N_\varepsilon(\imath))^{-1}Y_0(\imath)$. Employing the Schur complement, one can show that there exists a sufficiently small $\hat{\varepsilon}$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, (10.46) and (10.47) hold.

Now, we need to show that

$$\begin{pmatrix} X_\varepsilon(\imath) & I \\ I & Y_\varepsilon(\imath) \end{pmatrix} > 0. \tag{A.82}$$

By the Schur complement, it is equivalent to showing that

$$X_\varepsilon(\imath) - Y_\varepsilon^{-1}(\imath) > 0. \tag{A.83}$$

Substituting (A.80) and (A.81) into the left hand side of (A.83), we get

$$\begin{bmatrix} X_1(\imath) - Y_1^{-1}(\imath) & \varepsilon(X_2(\imath) + Y_1^{-1}(\imath)Y_2(\imath)Y_3^{-1}(\imath)) \\ \varepsilon(X_2(\imath) + Y_1^{-1}(\imath)Y_2(\imath)Y_3^{-1}(\imath))^T & \varepsilon(X_3(\imath) - Y_3^{-1}(\imath)) \end{bmatrix}. \tag{A.84}$$

The Schur complement of (10.56) is

$$\begin{bmatrix} X_1(\imath) - Y_1^{-1}(\imath) & 0 \\ 0 & X_3(\imath) - Y_3^{-1}(\imath) \end{bmatrix} > 0. \tag{A.85}$$

According to (A.85), we learn that

$$X_1(\imath) - Y_1^{-1}(\imath) > 0 \qquad \text{and} \qquad X_3(\imath) - Y_3^{-1}(\imath) > 0. \tag{A.86}$$

Using (A.86) and the Schur complement, it can be shown that there exists a sufficiently small $\hat{\varepsilon} > 0$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, (10.45) holds.

Next, employing (A.80) and (A.81), the controller's matrices given in (10.54) can be re-expressed as follows:

$$\begin{aligned} \mathcal{B}_i(\imath,\varepsilon) &= \left[Y_0^{-1}(\imath) - X_0(\imath)\right]\hat{B}_i(\imath) + \varepsilon\left[N_\varepsilon(\imath) - \tilde{X}(\imath)\right]\hat{B}_i(\imath) \triangleq \mathcal{B}_{0_i}(\imath) + \varepsilon\mathcal{B}_{\varepsilon_i}(\imath) \\ \mathcal{C}_i(\imath,\varepsilon) &= \hat{C}_i(\imath)Y_0^T(\imath) + \varepsilon\hat{C}_i(\imath)\tilde{Y}^T(\imath) \triangleq \mathcal{C}_{0_i}(\imath) + \varepsilon\mathcal{C}_{\varepsilon_i}(\imath). \end{aligned} \tag{A.87}$$

Substituting (A.80),(A.81) and (A.87) into (10.52) and (10.53), and pre-post multiplying (10.52) by $\begin{pmatrix} E_\varepsilon & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$, we respectively, obtain

$$\Psi_{11_{ij}}(\imath) + \psi_{11_{ij}}(\imath) \ \text{ and } \ \Psi_{22_{ij}}(\imath) + \psi_{22_{ij}}(\imath) \tag{A.88}$$

where the ε-independent linear matrices $\Psi_{11_{ij}}(\imath)$ and $\Psi_{22_{ij}}(\imath)$ are defined in (10.63) and (10.64), respectively, and the ε-dependent linear matrices are

$$\psi_{11_{ij}}(\imath) = \varepsilon \begin{pmatrix} \begin{pmatrix} A_i(\imath)\tilde{Y}^T(\imath) + \tilde{Y}(\imath)A_i^T(\imath) \\ +B_{2_i}(\imath)\mathcal{C}_{\varepsilon_j}(\imath) + \mathcal{C}_{\varepsilon_i}^T(\imath)B_{2_j}^T(\imath) \\ +\lambda_{\imath\imath}\hat{\tilde{Y}}(\imath) \end{pmatrix} & (*)^T & (*)^T \\ \tilde{C}_{1_i}(\imath)\tilde{Y}^T(\imath) + \tilde{D}_{12_i}(\imath)\mathcal{C}_{\varepsilon_j}(\imath) & 0 & (*)^T \\ \tilde{\mathcal{J}}^T(\imath) & 0 & -\tilde{\mathcal{Y}}(\imath) \end{pmatrix} \tag{A.89}$$

and

$$\psi_{22_{ij}}(\imath) = \varepsilon \begin{pmatrix} \begin{pmatrix} A_i^T(\imath)\tilde{X}^T(\imath) + \tilde{X}(\imath)A_i(\imath) \\ +\mathcal{B}_{\varepsilon_i}(\imath)C_{2_j}(\imath) + \ C_{2_i}^T(\imath)\mathcal{B}_{\varepsilon_j}^T(\imath) \\ +\sum_{k=1}^{s}\lambda_{\imath k}\hat{\tilde{X}}(k) \end{pmatrix} & (*)^T \\ \tilde{B}_{1_i}^T(\imath)\tilde{X}^T(\imath) + \tilde{D}_{21_i}^T(\imath)\mathcal{B}_{\varepsilon_j}^T(\imath) & 0 \end{pmatrix} \tag{A.90}$$

where $\tilde{\mathcal{J}}(\imath) = \left[\sqrt{\lambda_{1\imath}}\hat{\tilde{Y}}(\imath) \quad \cdots \quad \sqrt{\lambda_{(i-1)\imath}}\hat{\tilde{Y}}(\imath) \quad \sqrt{\lambda_{(i+1)\imath}}\hat{\tilde{Y}}(\imath) \quad \cdots \quad \sqrt{\lambda_{s\imath}}\hat{\tilde{Y}}(\imath)\right]$, $\tilde{\mathcal{Y}}(\imath) = \text{diag}\left\{\hat{\tilde{Y}}(1), \quad \cdots, \quad \hat{\tilde{Y}}(\imath-1), \quad \hat{\tilde{Y}}(\imath+1), \quad \cdots \quad, \hat{\tilde{Y}}(s)\right\}$, $\hat{\tilde{X}}(k) = \frac{\tilde{X}(k)+\tilde{X}^T(k)}{2}$ and $\hat{\tilde{Y}}(\imath) = \frac{\tilde{Y}(\imath)+\tilde{Y}^T(\imath)}{2}$. Note that the ε-dependent linear matrices tend to zero when ε approaches zero.

Employing (10.59)-(10.62) and knowing the fact that for any given negative definite matrix $\mathcal{W}$, there exists an $\varepsilon > 0$ such that $\mathcal{W} + \varepsilon I < 0$, one can show that there exists a sufficiently small $\hat{\varepsilon} > 0$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, (10.48)-(10.51) hold. Since (10.45)-(10.51) hold, using Lemma 9, the inequality (5.5) holds. ∎

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